Structural Breaks in U.S. Macroeconomic Time Series: A Bayesian Model Averaging Approach

Online Appendix

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1 Introduction

This appendix describes additional Monte Carlo experiments conducted for the Bayesian model averaging (BMA) procedure proposed in Check and Piger (2020), "Structural Breaks in U.S. Macroeconomic Time Series: A Bayesian Model Averaging Approach." These Monte Carlo experiments investigate additional data generating processes (DGP) over those considered in Check and Piger (2020), and in particular focus on DGPs that are mis-specified in some dimension from the finite-order autoregressive (AR) model with structural breaks assumed by their BMA procedure. For each DGP, 100 series of size $T = 226$ are generated, and the BMA procedure assuming an autoregressive (AR) process with parameter breaks is applied to each series. All details of the implementation of the BMA procedure are as described in Section 4 of Check and Piger (2020). In the following we describe each of the

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DGPs considered and report the results of the associated Monte Carlo experiment. The results presented are averages across the 100 Monte Carlo simulations.

2 ARMA Data Generating Process

In this section we describe results from a DGP that is an ARMA(1,1) with structural breaks in intercept:

$$y_t = \alpha_t + \phi y_{t-1} + \theta \varepsilon_{t-1} + \varepsilon_t, \quad t = 1, 2, \ldots, T$$
$$\varepsilon_t \sim i.i.d.\ N\left(0, h^{-1}\right)$$

where α_t undergoes two structural breaks approximately 1/3 and 2/3 through the sample. The size of the structural breaks is calibrated as for the "Intercept Large" case described in Section 4 of Check and Piger (2020). We set the autoregressive and moving average parameters to $\phi = 0.3$ and $\theta = 0.3$ respectively. Finally, the value of h is set to match the unconditional variance of y_t for the "Intercept Large" case described in Section 4 of Check and Piger (2020). All parameter values used in the DGP are detailed in the top panel of Table 1.

The results of this Monte Carlo experiment are detailed in the second columns of Tables 2-4. Table 2 shows the posterior inclusion probability for alternative numbers of autoregressive lags. For the ARMA DGP, the BMA procedure selects the first lag with 100% posterior probability, but places very low posterior probability on higher order lags. As this ARMA model has an AR(∞) representation with non-trivial AR lags beyond order one, this suggests that the BMA procedure underfits the dynamics of the true DGP. However, from Tables 3-4, we see that this underfitting does not affect the ability of the BMA procedure to detect the number and nature of structural breaks. Specifically, Table 3 shows that the BMA procedure places 92% posterior probability on the correct number of structural breaks, while Table 4 shows that this posterior probability is placed nearly entirely on structural breaks in intercept, with almost no evidence found for structural breaks in any other parameters.

3 Markov-Switching Data Generating Process

In this section we describe results from a DGP that is an AR(1) process with Markov-switching intercept:

$$y_t = \alpha_{S_t} + \phi y_{t-1} + \varepsilon_t, \quad t = 1, 2, \ldots, T$$

$$\varepsilon_t \sim i.i.d.\ N\left(0, h^{-1}\right)$$

where $\alpha_{S_t} = \alpha_0(1 - S_t) + \alpha_1(S_t)$ and $S_t \in \{0, 1\}$ follows a two state Markov process with transition probabilities $p_{00} = \Pr(S_t = 0|S_{t-1} = 0)$ and $p_{11} = \Pr(S_t = 1|S_{t-1} = 1)$. To set the parameters of the model, we use the values reported in Table 4.1 of Kim and Nelson (1999) from their fitting of this model to U.S. real GDP growth. In Kim and Nelson (1999), the Markov-switching fits U.S. recession and expansion dates closely, giving $S_t = 0$ the interpretation of a normal growth expansion regime and $S_t = 1$ the interpretation of a low growth recession regime. In the DGP, there are no structural breaks in any of the model parameters. All parameter values used in the DGP are detailed in the middle panel of Table 1.

The results of this Monte Carlo experiment are detailed in the third columns of Tables 2-4. Table 2 shows the posterior inclusion probability for alternative numbers of autoregressive lags. For the Markov-switching DGP, the BMA procedure places 19% posterior probability on the first lag, and places very low posterior probability on higher order lags. In the true DGP, the AR(1) parameter is only 0.1, so the relatively low posterior probability placed on this lag is not surprising. From Table 3, we see that the BMA procedure detects the absence of structural breaks very accurately. Specifically, Table 3 shows that the BMA procedure places 99% posterior probability on the model with no structural breaks.

4 Stochastic Volatility Data Generating Process

In this section we describe results from a DGP that is an AR(1) process with stochastic volatility:

$$y_t = \alpha + \phi y_{t-1} + \varepsilon_t$$

$$\varepsilon_t \sim i.i.d.\ N\left(0, \exp\left\{h_t\right\}\right)$$

$$h_t = h_{t-1} + \eta_t$$

$$\eta_t \sim i.i.d.\ N(0, \gamma^2)$$

We parameterize the volatility process to be persistent with relatively small variance changes. This produces a volatility process that drifts slowly over time, in order to separate the stochastic volatility process conceptually from that of a structural break process. In the DGP, there are no structural breaks in any of the model parameters. All parameter values used in the DGP are detailed in the bottom panel of Table 1.

The results of this Monte Carlo experiment are detailed in the fourth columns of Tables 2-4. Table 2 shows the posterior inclusion probability for alternative numbers of autoregressive lags. For the stochastic volatility DGP, the BMA procedure places 100% posterior probability on the first lag, and places zero posterior probability on higher order lags. Thus, it places 100% posterior probability on the true AR(1) process. From Table 3, we see that the BMA procedure also detects the absence of structural breaks very accurately. Specifically, Table 3 shows that the BMA procedure places 97% posterior probability on the model with no structural breaks.

Table 1
Monte Carlo Data Generating Processes — Misspecified Models

	Regime 1	Regime 2	Regime 3
ARMA with Intercept Breaks			
α_t	1.4	0.0	-1.4
ϕ	0.3	0.3	0.3
θ	0.3	0.3	0.3
h^{-1}	0.81	0.81	0.81
Markov Switching			
α_0	0.92		—
α_1	-0.21		—
ϕ	0.1		—
h^{-1}	0.64		—
p_{00}	0.90		—
p_{11}	0.76		—
Stochastic Volatility			
α	0.0	—	—
ϕ	0.3	—	—
γ	0.01	—	—
$exp(h_0)$	1.0	—	—

Table 2
Posterior Inclusion Probabilities for Autoregressive Lags

Autoregressive Lag	ARMA	Markov Switching	Stochastic Volatility
1	**100**	**19**	**100**
2	3	0	1
3	0	0	0
4	0	0	0
5	0	0	0
6	0	0	0

Notes: Results shown are averages across Monte Carlo simulations. Bold type in a column indicates an autoregressive lag that is present in the DGP for that column.

Table 3
Posterior Probability for Alternative Numbers of Structural Breaks

Number of Breaks	ARMA	Markov Switching	Stochastic Volatility
0	0	**99**	**97**
1	8	1	2
2	**92**	0	1
3	0	0	0
4	0	0	0
> 4	0	0	0

Notes: Results shown are averages across Monte Carlo simulations. Bold type in a column indicates the true number of structural breaks present in the DGP for that column.

Table 4
Mean Number of Structural Breaks in Individual Model Parameters

Parameter	ARMA	Markov Switching	Stochastic Volatility
Intercept	**1.92 (2)**	0.01 (0)	0.03 (0)
AR(1)	0.01 (0)	0.00 (0)	0.01 (0)
AR(2)	0.00 (0)	0.00 (0)	0.00 (0)
AR(3)	0.00 (0)	0.00 (0)	0.00 (0)
AR(4)	0.00 (0)	0.00 (0)	0.00 (0)
AR(5)	0.00 (0)	0.00 (0)	0.00 (0)
AR(6)	0.00 (0)	0.00 (0)	0.00 (0)
Variance	0.03 (0)	0.02 (0)	0.01 (0)

Notes: Results shown are averages across Monte Carlo simulations. Entries in parentheses give the true number of structural breaks for the respective parameter and DGP. Bold type in a column indicates a parameter that experienced structural breaks in the DGP for that column.

References

Kim, C.-J. and C. R. Nelson (1999). *State-Space Models with Regime Switching.* Cambridge, MA: The MIT Press.