The Importance of Nonlinearity in Reproducing Business Cycle Features

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ABSTRACT: This paper considers the ability of simulated data from linear and nonlinear time-series models to reproduce features in U.S. real GDP data related to business cycle phases. We focus our analysis on a number of linear ARIMA models and nonlinear Markov-switching models. To determine the timing of business cycle phases for the simulated data, we present a model-free algorithm that is more successful than previous methods at matching NBER dates and associated features in the postwar data. We find that both linear and Markov-switching models are able to reproduce business cycle features such as the average growth rate in recessions, the average length of recessions, and the total number of recessions. However, we find that Markov-switching models are better than linear models at reproducing the variability of growth rates in different business cycle phases. Furthermore, certain Markov-switching specifications are able to reproduce high-growth recoveries following recessions and a strong correlation between the severity of a recession and the strength of the subsequent recovery. Thus, we conclude that nonlinearity is important in reproducing business cycle features.

1. Introduction

In recent years, there has been an explosion of research devoted to developing nonlinear time-series models of the univariate dynamics in measures of U.S. economic activity such as real GDP. Much of the work in this research agenda augments linear ARIMA models in ways that capture regime shifts in mean, variance, or dynamic propagation. A particularly popular approach, due to Hamilton (1989), is to model the regimes as unobserved, but as following a Markov process. Hamilton's original specification allowed for two regimes in the mean of an autoregressive model of U.S. real GNP growth. Hansen (1992) extended Hamilton's Markov-switching model to allow for regime switching in parameters other than the mean growth rate, namely the residual variance and autoregressive parameters. The Hamilton model has also been modified to allow for more than two regimes by Boldin (1996) and Clements and Krolzig (1998).

An important question is whether or not these Markov-switching models are superior to their linear alternatives in describing U.S. real GDP. One way to approach this question is to conduct statistical tests of the null hypothesis of a linear ARIMA model against a particular nonlinear alternative. Such analysis has yielded mixed evidence regarding the value-added of nonlinear models. For example, Garcia (1998) and Hansen (1992) were unable to reject a linear model in favor of the Hamilton model of real GNP growth. However, Hansen (1992) and Kim, Morley and Piger (2005) were able to reject linearity in favor of extended versions of the Hamilton model.

An alternative approach for comparing nonlinear and linear models of real GDP is to evaluate the ability of such models to reproduce certain characteristics of the sample data, such as business cycle features. In particular, do simulated data from a nonlinear time-series model behave more like the sample data than simulated data from a linear model? Such a comparison seems natural, as many of the nonlinearities explored for U.S. real GDP have been motivated as related to certain characteristics of the sample data, especially those related to the business cycle. Past studies that consider the ability of linear and nonlinear models to reproduce business cycle features include Hess and Iwata (1997), Harding and Pagan (2002), Galvão (2002), Clements and Krolzig (2004), and Kim, Morley, and Piger (2005).

The analysis of a model's ability to reproduce business cycle features requires a definition of the business cycle. In the spirit of Burns and Mitchell (1946), the practice in the literature on business cycle features is to define the business cycle as a series of distinct phases in economic activity, with the phases corresponding to recession and expansion. The timing of the phases is measured using a model-free algorithm that identifies peaks and troughs. Based on these dates, standard business cycle features, such as the average cumulative growth experienced during business cycle phases and the average length of a phase, are computed for the sample data. Then, using the algorithm to identify peaks and troughs, corresponding business cycle features are computed for simulated data from a model in order to evaluate its ability to reproduce features in the sample data.[1]

The work of Hess and Iwata (1997) and Harding and Pagan (2002) found little or no value-added for nonlinear models, including Markov-switching models, over linear models. Indeed, for several business cycle features, the nonlinear models performed worse.[2]
Galvão (2002) expanded the class of Markov-switching models considered by Harding and Pagan (2002) to include models with a third regime that is meant to capture a high growth recovery phase following the end of recessions. She finds that such models are better able than

[1] This evaluation can be thought of as an example of an encompassing test of model specification. See Breunig, Najarian, and Pagan (2003) for a general discussion of encompassing tests for Markov-switching models.
[2] It is important to note that Hess and Iwata (1997) used a nonstandard definition of the business cycle. In particular, they labeled any switch between positive and negative growth, no matter how short-lived, to be a business cycle

linear models to capture the apparent concave shape of U.S. real GDP during expansions.[3] Clements and Krolzig (2004) consider multivariate extensions of two-regime Markov-switching models and find that such models provide little improvement over linear models in capturing business cycle features. Like Galvão (2002), Kim, Morley, and Piger (2005) find that models with a high growth recovery phase perform better than linear models in capturing business cycle features.

In this paper, we revisit the relative ability of nonlinear vs. linear models to reproduce business cycle features in postwar U.S. real GDP data, focusing on a number of linear ARIMA models and Markov-switching models. The Markov-switching models are the original two-regime switching-mean model of Hamilton (1989), the three-regime switching-mean model of Boldin (1996), and the regime-switching "bounceback" model of Kim, Morley, and Piger (2005). The "bounceback" model allows the mean growth rate of the time series to depend on lagged regime outcomes, which, like the three-regime model, allows for high growth in the period immediately after a recession. However, unlike the three-regime model, the "bounceback" model predicts that more severe recessions will be followed by more rapid growth in the recovery.

In capturing nonlinearity, we focus on Markov-switching models, while acknowledging that there are many other nonlinear models designed to capture business cycle features. For example, there is a large literature on models that allow dynamics to change when an observed indicator variable exceeds a give threshold, including Beaudry and Koop (1993), Tiao and Tsay (1994), Potter (1995), Pesaran and Potter (1997), van Dijk and Franses (1999), and Öcal

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turning point. For U.S. real GDP, their methodology identifies twice as many turning points as reported by the NBER.

[3] Sichel (1994) provides an extensive analysis of the presence and sources of a high growth phase following recessions in U.S. real GDP.

and Osborn (2000). However, we focus on Markov-switching models for the following reasons: First, there is a transparent link between the form of nonlinearity in such models and the business cycle. Specifically, the nonlinearity is driven by regime changes that appear to match closely with business cycle turning points, with the models being conditionally linear within regimes. Second, compared to linear models, Markov-switching models are close substitutes to other nonlinear models in terms of their ability to forecast (Clements and Krolzig, 1998) and their ability to capture nonlinear characteristics such as "deepness" and "steepness" (Clements and Krolzig, 2003). Third, Markov-switching models have been the focus of much of the controversy over the importance of nonlinearity in reproducing business cycle features, as is evident in the debate between Harding and Pagan (2003) and Hamilton (2003). Fourth, the statistical support for the Markov-switching "bounceback" model in Kim, Morley, and Piger (2005) suggests that regime switching models may provide an effective way to capture nonlinearities in the data.

To obtain business cycle peak and trough dates in both the sample and simulated data, we use a modified version of the algorithm presented in Harding and Pagan (2002). Our modified version of the algorithm is more successful than previous methods at matching the NBER dates and associated features in the postwar data. Using the peak and trough dates from the algorithm, we define a series of standard business cycle features, including mean and standard deviation of growth rates observed during expansion and recession phases and the mean and standard deviation of the length of phases.[4] In addition, we divide the expansion phase into a recovery phase, defined as the four quarters following the end of a recession, and a mature expansion phase, defined as the remainder of the expansion, and compute business cycle features for each of these phases separately. We also compute the correlation between the cumulative growth

[4] We are interested in the standard deviation features because they capture the substantial heterogeneity of historical business cycles.

observed during recessions and the ensuing cumulative growth observed in the subsequent recovery phase, a business cycle feature that was central to Milton Friedman's (1964, 1993) analysis of U.S. business cycles.[5]

Our analysis yields four main conclusions: First, consistent with past studies, Markov-switching models do not appear to systematically improve on the ability of linear models to reproduce certain features of postwar business cycles such as the average growth rate in a recession phase, the average length of a recession, and the overall number of recessions. Indeed, both linear and Markov-switching models are adequate for this task. Second, the regime-switching models seem to improve on linear models in terms of the variability of growth rates observed for different business cycle phases. Third, consistent with Galvão (2002) and Kim, Morley, and Piger (2005), the three-regime and "bounceback" models, both of which have a mechanism for capturing a high-growth recovery phase following recessions, dominate linear models at reproducing the pattern of postwar expansions. Fourth, the "bounceback" model dominates linear models and the other Markov-switching models at capturing the strong correlation in the sample data between the severity of a recession and strength of the subsequent recovery. Taken together, the results lead us to conclude that certain Markov-switching specifications can yield substantial improvements over linear models in reproducing business cycle features.

An important question is whether the performance of the nonlinear models in reproducing characteristics of the sample data is being influenced by something other than simply capturing nonlinear dynamics related to the business cycle. In particular, regime switching in the mean can generate patterns that look similar to a structural break in volatility or other forms of

[5] See also Wynne and Balke (1992, 1996).

heteroskedasticity that may be present in the sample data. Indeed, there is strong evidence that a structural break in the volatility of U.S. real GDP growth occurred sometime around 1984.[6] To examine this issue, we consider the business cycle features implied by linear and nonlinear models that account for a structural break. Also, for the regime-switching models, we consider specifications that allow for switching in the variance, as in Clements and Krolzig (1998, 2003). However, we find that accounting for a structural break in volatility and for regime switching in variance has little impact on our results.

The remainder of this paper is organized as follows: Section 2 describes the algorithm we use to establish business cycle turning points in U.S. real GDP and compares these dates to those established by the National Bureau of Economic Research (NBER) Business Cycle Dating Committee. Section 3 defines the business cycle features we consider in this paper and documents these features for the sample data. Section 4 evaluates the ability of the competing linear and Markov-switching models to reproduce these features. Section 5 concludes.

2. An Algorithm for Establishing Business Cycle Turning Points

Given the definition of the business cycle adopted in this paper, it seems natural to use the NBER business cycle peak and trough dates for calculating business cycle features. However, the NBER chronology is only available for the sample data, not for the simulated data from the time-series models. Thus, to establish turning points in the sample data and simulated data in a consistent fashion, we need to use a formal procedure capable of mimicking the NBER decision-making process reasonably well. To this end, Harding and Pagan (2002) use a quarterly

[6] There is a vast literature on the structural break in volatility, including Niemera and Klein (1994), Kim and Nelson (1999), McConnell and Perez-Quiros (2000), Stock and Watson (2002), Sensier and van Dijk (2004), Kim, Nelson, and Piger (2004), and Kim, Morley, and Piger (2004).

version of the Bry and Boschan (1971) algorithm for establishing business cycle turning points. This algorithm, named the BBQ algorithm by Harding and Pagan (2002), has the following basic steps:

1) Using the log level of U.S. quarterly real GDP, denoted y_t, establish candidate dates of peaks and trough as local maxima and minima in the data. In particular define a peak at time *t* if:

$$y_{t-2} - y_t < 0; \quad y_{t-1} - y_t < 0; \quad y_{t+1} - y_t < 0; \quad y_{t+2} - y_t < 0,$$

   and a trough at time *t* if:

$$y_{t-2} - y_t > 0; \quad y_{t-1} - y_t > 0; \quad y_{t+1} - y_t > 0; \quad y_{t+2} - y_t > 0.$$

2) Censor the turning points to ensure that peaks and troughs alternate. In the case of two consecutive peaks (troughs), eliminate the peak (trough) with the lower (higher) value of y_t.

3) Censor the turning points to ensure that each business cycle phase (peak-to-trough and trough-to-peak) lasts a minimum of two quarters, while each complete business cycle (peak-to-peak and trough-to-trough) lasts a minimum of five quarters.

Table 1 reports the peak and trough dates established by the NBER for the sample period 1948:Q4 – 2003:Q2, along with the dates established by the BBQ algorithm applied to quarterly U.S. real GDP. It is clear that the BBQ algorithm does a fairly good job of replicating the NBER dates. Specifically, of the nineteen NBER turning points in the sample, the algorithm matches the NBER date exactly in nine cases. For nine of the remaining ten cases, the date established by the algorithm is within one quarter of that established by the NBER. However, the algorithm does make some systematic errors. For example, for the ten turning points for which the dates do not match exactly, the algorithm shifts the timing of recessions forward in time. That is, the dates for both peaks and troughs are too early compared to the NBER dates.

<Table 1 Here>

These systematic errors suggest that the BBQ algorithm might be improved by modifying the first step of the algorithm above. In particular, this step can be generalized for a modified BBQ (MBBQ) algorithm as follows. Define a peak at time *t* if:

$$y_{t-2} - y_t < \alpha_1; \;\; y_{t-1} - y_t < \alpha_1; \;\; y_{t+1} - y_t < \alpha_2; \;\; y_{t+2} - y_t < \alpha_2,$$

and a trough at time *t* if:

$$y_{t-2} - y_t > \alpha_3; \;\; y_{t-1} - y_t > \alpha_3; \;\; y_{t+1} - y_t > \alpha_4; \;\; y_{t+2} - y_t > \alpha_4.$$

That is, we allow the threshold parameters that signal turning points to differ from zero. We also allow these thresholds to vary from peak to trough and on different sides of turning points. To determine the values of the $\alpha_i's$, we search over values in a grid close to zero, namely $\alpha_i \in (-0.005, 0.005)$. For each possible combination of the $\alpha_i's$ in the grid, we compute the mean squared error (MSE) as follows:

$$MSE(\alpha_i) = \sqrt{\frac{\sum_{t=1}^{T}(MBBQ_t(\alpha_i) - NBER_t)^2}{T}} \; ,$$

where $NBER_t = 1$ if quarter t is an NBER recession quarter and $NBER_t = 0$ otherwise, while $MBBQ_t = 1$ if quarter t is a recession quarter according to the MBBQ algorithm and $MBBQ_t = 0$ otherwise. We then choose those values of the $\alpha_i's$ that minimize $MSE(\alpha_i)$. In the case of ties, we choose the $\alpha_i's$ that are closest to zero, as measured by $\sum_{i=1}^{4}|\alpha_i|$.

Table 1 also reports the peak and trough dates established by the MBBQ algorithm for the chosen values of the $\alpha_i's$, $\alpha_1 = -0.002$, $\alpha_2 = 0.002$, $\alpha_3 = 0.002$, $\alpha_4 = -0.002$. The MBBQ algorithm provides a substantial improvement over the BBQ algorithm in matching the NBER dates. In particular, of the nineteen turning points, the MBBQ algorithm matches fourteen exactly, versus ten for the BBQ algorithm. Of the remaining five, all are within one quarter of the NBER dates. More importantly, as discussed in the next section, the business cycle features produced using the MBBQ algorithm are generally closer to those using the NBER dates than are those using the BBQ algorithm.

3. Business Cycle Features in U.S. Real GDP Data

Given the peak and trough dates for U.S. real GDP, we define four business cycle phases: 1) Recession, defined as the quarter following a peak date to the subsequent trough date, 2) Expansion, defined as the quarter following a trough date to the subsequent peak date, 3) Recovery, defined as the first four quarters of the Expansion phase, and 4) Mature Expansion, defined as the remaining quarters of an Expansion phase following the Recovery phase.

Given these phases for a given realization of data, we then define the following business cycle features:

- Number of business cycle peaks.
- Average of recession and expansion phase lengths.
- Standard deviation of recession and expansion phase lengths.
- Average of annualized quarterly growth rates in recession phases, expansion phases, recovery phases, and mature expansion phases.
- Standard deviation of annualized quarterly growth rates in recession phases, expansion phases, recovery phases, and mature expansion phases.
- Correlation between the cumulative decline during a recession and the cumulative growth in the subsequent recovery phase.

Table 2 presents values of these business cycle features for quarterly U.S. real GDP data from 1948:Q4 – 2003:2. The features are calculated given turning points established by the NBER, the BBQ algorithm, and the MBBQ algorithm. In general, the features are very similar for each dating method. However, the MBBQ is better able than the BBQ to match the average growth rate in recoveries based on NBER dates. Thus, for the remainder of the paper, we concentrate on the results for the MBBQ algorithm.

<Table 2 Here>

Several items in Table 2 are of interest. First of all, there are substantial differences in the average annualized quarterly growth rates observed over different business cycle phases. Recessions on average correspond to substantial declines in economic activity, averaging around

2.5% per quarter, while expansion phases correspond to substantial gains, averaging around 4.5% per quarter. Expansion phases can also usefully be divided between the Recovery and Mature Expansion phase, with the average growth rates in the Recovery phase nearly 50% larger than in the Mature Expansion phase. Secondly, as is well known, the average length of recession phases is much shorter than expansion phases. Third, there is substantial variation in quarterly growth rates during business cycle phases, with this variability higher in the Recovery phase than in the Mature Expansion and Recession phase. Such high variability is also observable in the length of business cycle phases. For example, while the average length of an Expansion phase is roughly 20 quarters, the standard deviation of this length is nearly 13 quarters. Finally, as noted by Milton Friedman (1964, 1993), there is a strong negative correlation between the cumulative growth in a Recession phase and the cumulative growth in the subsequent Recovery phase. The size of this correlation is striking at -0.75. In the next section, we evaluate the extent to which linear and Markov-switching time-series models of real GDP can reproduce these business cycle features.

4. Business Cycle Features in Simulated Data from Time-Series Models

4.1 Model Description and Estimation

We consider nine different models of U.S. real GDP growth rates, six which are in the linear ARMA class and three which are nonlinear models with Markov-switching parameters. The models have the following general representation:

$$\phi(L)\left(\Delta y_t - \mu_t\right) = \theta(L)\varepsilon_t \, , \tag{1}$$

where $\phi(L) = 1 - \phi_1 L - \cdots - \phi_k L^k$, $\theta(L) = 1 + \theta_1 L + \cdots + \theta_r L^r$, $\varepsilon_t \sim N(0, \sigma^2)$, and the specification for μ_t depends on whether the model is linear or nonlinear.

The linear ARMA models have a fixed mean:

$$\mu_t = \mu \,. \tag{2}$$

In terms of the dynamics, we consider AR(1), AR(2), MA(1), MA(2), ARMA(1,1) and ARMA(2,2) specifications. We consider this large number of linear specifications to make it clear that any findings in terms of reproducing business cycle features are the result of linearity rather than a particular linear model specification.

For the nonlinear models, we set $\phi(L) = 1$ and $\theta(L) = 1$ (i.e., $k = r = 0$). Thus, the nonlinear models do not generate any dynamics through linear propagation. Instead, all dynamics arise from the regime-switching parameters in the mean. This helps us to conduct a transparent evaluation of the relative importance of linearity and nonlinearity in reproducing business cycle features.[7]

The first regime-switching model is based on the original two-regime switching-mean model of Hamilton (1989). For this model, the mean has the following specification:

$$\mu_t = \mu_1 I(S_t = 1) + \mu_2 I(S_t = 2)\,, \tag{3}$$

[7] Of course, the preferred model in terms of a statistical comparison may very well contain both linear and nonlinear dynamics. Our objective here is not to identify a particular preferred model, but instead to evaluate whether nonlinearity is likely to be a component of this model.

where the indicator function $I(S_t = j)$ is equal to 1 if $S_t = j$ and zero otherwise and the state variable $S_t = \{1,2\}$ follows a Markov process with fixed transition probabilities given by

$$P(S_t = 1 \mid S_{t-1} = 1) = p_{11},$$

$$P(S_t = 2 \mid S_{t-1} = 2) = p_{22},$$

where S_t is normalized by $\mu_1 > \mu_2$. Hamilton (1989) found that the estimated values of S_t from this model correspond to NBER business cycle phases, and that μ_1 and μ_2 capture the tendency of real GDP to grow during expansions and decline during recessions.[8]

The second regime-switching model is based on the three-regime switching-mean model of Boldin (1996).[9] For this model, the mean has the following specification:

$$\mu_t = \mu_1 I(S_t = 1) + \mu_2 I(S_t = 2) + \mu_3 I(S_t = 3), \qquad (4)$$

where the state variable $S_t = \{1,2,3\}$ again follows a Markov process, in this case with a fixed transition probability matrix:

$$\begin{bmatrix} p_{11} & p_{12} & 0 \\ 0 & p_{22} & p_{23} \\ p_{31} & 0 & p_{33} \end{bmatrix}$$

[8] The ability of Hamilton's (1989) model to capture NBER business cycle phases using recent vintages of data depends on the inclusion of linear dynamics in the model. For the case without linear dynamics, which is the specification considered in this paper, the estimated regimes closely match NBER business cycle phases. See Chauvet and Piger (2003) on this point.

[9] See Sichel (1994) on the motivation for a three-regime model. Also, Clements and Krolzig (1998) present a three-regime Markov-switching model with a slightly different specification.

That is, the regime sequence is restricted to follow the pattern $\{S_t\} = \cdots 1 \to 2 \to 3 \to 1 \cdots$. Given the normalization $\mu_1 > \mu_2$, Boldin finds that $S_t = 2$ corresponds to NBER recessions, while the $S_t = 3$ regime captures the tendency of output, evident in Table 2, to undergo a high-growth recovery phase following NBER recessions. As pointed out in Clements and Krolzig (2003), the three-regime model can capture business cycle asymmetries such as steepness and deepness, while the two-regime model can only capture deepness.

Finally, the "bounceback" model of Kim, Morley and Piger (2005) has the following form for μ_t:

$$\mu_t = \mu_1 I(S_t = 1) + \mu_2 I(S_t = 2) + \lambda I(S_t = 1)\sum_{j=1}^{m} I(S_{t-j} = 2), \qquad (5)$$

where the state variable S_t is the same as for Hamilton's (1989) model. Kim, Morley and Piger find that $S_t = 2$ again corresponds to NBER recessions and that $m = 6$ and $\lambda > 0$, implying a high growth recovery phase in the first six quarters following the end of recessions.[10] Further, and unlike the three-regime model in (4), the strength of this high growth recovery phase is related to the severity of the previous recession, as measured by its length up to six quarters.

<Table 3 Here>

Each model is estimated via maximum likelihood for U.S. real GDP growth rates over the period 1949:Q1 – 2003:Q2. Table 3 reports the estimates for the linear models. The ARMA(2,2) has a considerably higher likelihood value than the other models, although the Schwarz criterion

[10] The indicator variable for the current state in the third term of the mean equation represents a change from the specification in Kim, Morley, and Piger (2005). In the current specification, the third term only affects the growth of

favors the AR(1) model. Table 4 reports the estimates for the regime-switching models. The estimates are similar to those found in other papers. In every case, including the two-regime model, the parameter estimates are suggestive of switching associated with business cycle regimes. The "bounceback" model has the highest likelihood value and is favored by the Schwarz criterion over all the other models, including the linear models presented in Table 3.[11]

<Table 4 Here>

Next, we use the estimated parameters to simulate artificial GDP series from 1948:Q4 – 2003:Q2, using the actual value of real GDP from 1948:Q4 as an initial value. For each model, we perform 10,000 simulations, saving the business cycle features for each simulation.

4.2 Business Cycle Features from Linear Models

Table 5 reports percentiles of sample values for business cycle features in terms of the simulated distributions of these features for the linear models. These percentiles provide a sense of how likely a model could have produced a given sample value for a feature. Any percentiles less than 0.10 or greater than 0.90 are in bold, denoting that it was unlikely that a sample value could have arisen from a time-series process characterized by a given model. Meanwhile, to give a sense of whether a percentile is driven by closeness of the distribution in matching the sample feature or by a large dispersion of the simulated distribution, we also consider the difference between the sample value from Table 2 and the median of a given distribution. This difference is reported in parentheses.

<Table 5 Here>

GDP after a recession is over, making the model directly comparable to the three state model. We thank Jim Hamilton for the suggestion of this specification.

[11] It should be noted that having the highest likelihood is no assurance of being the best model in terms of fitting business cycle features. For example, as discussed below, we consider more general models that explicitly allow for

The results in Table 5 make it clear that the linear ARMA models have some successes and some failures in reproducing the business cycle features in the sample data. Beginning with the successes, and perhaps surprisingly, the linear models do a reasonably good job of matching the average growth rate during a recession. The sample value of -2.44% is somewhat below the median average growth rate produced by each model, but the simulated distributions for this feature easily subsume the sample value. The linear models also do a good job of matching the number of business cycle peaks, as well as the average length of recessions and expansions. The linear models are also able to generate variability of Recession and Expansion phase lengths that are consistent with the sample data.

The most notable failure of the linear models is with regards to features related to the Recovery phase. For example, the linear models cannot generate average growth in the Recovery phase that matches the sample data. Essentially none of the 10,000 simulated values for each model is as high as the sample value of 6.67%, which is about 2 percentage points above the medians of the simulated average growth rates in the recovery phase. Also, the linear models are unable to generate any meaningful negative correlation between the cumulative growth in the Recession phase and the cumulative growth in the subsequent Recovery phase.

The linear models also have difficulty generating growth rate variability that matches the sample data in Recession phases, Recovery phases, and Mature Expansion phases. In particular, the median standard deviations of the growth rates in Recession and Recovery phases are too low, while the median standard deviations of growth rates in the Mature Expansion phase are too high.

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heteroskedasticity. These models always have higher likelihood values than their homoskedastic counterparts. However, interestingly, they do not always perform better in terms of reproducing business cycle features.

The conclusion is that linear models appear to be inconsistent with certain features of the data related to business cycles. Taken together, the large number of extreme percentiles for the sample features suggest that it is highly unlikely that the data arose from a linear data generating process. The next question is whether this failure is a failure of linearity or of time series modeling more generally. That is, can nonlinear models do better at reproducing business cycle features?

4.3 Business Cycle Features from Regime-Switching Models

Table 6 reports percentiles of sample values for business cycle features in terms of the simulated distributions of these features for the regime-switching models. Interestingly, the two-regime model has little success at improving on the linear models. In particular, it fails for many of the same features as the linear models, namely those related to recovery growth rates. This evidence is consistent with the results in Hess and Iwata (1997), Harding and Pagan (2002), and Galvão (2002), and is perhaps not surprising given the scant evidence that has been found in favor of the basic two-regime model over linear alternatives using statistical tests (e.g. Hansen, 1992; Garcia, 1998).

<Table 6 Here>

Consistent with Galvão (2002) and Kim, Morley, and Piger (2005), we find that the three-regime model does improve on the ability of the linear models to generate high growth in the Recovery phase. In particular, as much as 15% of the simulated values for the average growth in Recovery phases fall above the sample value of 6.67%, which is only about 1 percentage point above the median simulated value. Also, the three-regime model is better able to capture the variability of growth rates during Recovery phases than the linear models. Finally, the three-regime model generates a negative median correlation between cumulative growth in the Recession phase and the Recovery phase. However, only 6% of the simulated correlations fall below the sample value of -0.75.

The best performance of the regime-switching models comes from the "bounceback" model. Indeed, with only one exception, that being the variability of growth rates during Mature Expansion phases, the sample value for each feature falls between 10% and 90% of the corresponding simulated distribution for the "bounceback" model. The model displays a clear improvement over all of the other models, including the three-regime model, in generating a median value for the average growth rate in Recovery phases close to the sample value of 6.67%. The difference is less than half of a percentage point. Finally, the model generates a median correlation between cumulative growth in the Recession phase and the subsequent Recovery phase that is much closer to the sample value of -0.75.

The conclusion is that the three-regime and "bounceback" models are able to generate improvements over linear models in reproducing certain business cycle features, with the "bounceback" model displaying the most improvement. The models achieve this gain without noticeable deterioration in reproducing other features. However, it is not the case that any regime-switching model is better than the linear models. In particular, consistent with Galvão (2002) and Kim, Morley, and Piger (2005), it appears to be very important to have a nonlinear model that captures high growth in the Recovery phase. Further, linking the severity of output declines in the Recession phase to the strength of growth in the Recovery phase is also important to generate business cycle patterns that match the sample data.

4.4 Business Cycle Features and Heteroskedasticity

As discussed above, there is strong evidence that a structural reduction in the volatility of U.S. real GDP growth occurred sometime around 1984. One possible concern is that the linear models are set up to fail because they do not account for this structural break. The presence of a structural break in volatility could account for a higher variability of growth rates in recessions and recoveries that cannot easily be generated by the linear models in Table 3. Meanwhile, the Markov-switching models in Table 4 may perform better because they can potentially proxy for the structural break or other forms of heteroskedasticity, rather than simply capturing nonlinearity related to the business cycle. To address this issue, we consider the ability of models with a structural break in volatility to reproduce business cycle features. Formally, the model error term is specified as

$$\varepsilon_t \sim N(0, \sigma_t^2)\,, \tag{6}$$

$$\sigma_t^2 = \sigma_{A,t}^2 * (1 - D_t) + \sigma_{B,t}^2 * D_t\,,$$

where D_t is zero before the first quarter of 1984 and one thereafter. For the linear models, the residual variance only changes at the structural break date, so that

$$\sigma_{i,t}^2 = \sigma_i^2,\ i = A, B\,. \tag{7}$$

For the Markov-switching models, we also consider specifications in which the residual variance changes over business cycle regimes. That is, for the two-regime and "bounceback" models, the residual variance is

$$\sigma_{i,t}^2 = \sigma_{i,1}^2 I(S_t = 1) + \sigma_{i,2}^2 I(S_t = 2),\ i = A, B\,. \tag{8}$$

Meanwhile, for the three-regime model, the residual variance is

$$\sigma_{i,t}^2 = \sigma_{i,1}^2 I(S_t = 1) + \sigma_{i,2}^2 I(S_t = 2) + \sigma_{i,3}^2 I(S_t = 3),\ i = A, B\,. \tag{9}$$

<Table 7 Here>

Tables 7 and 8 report the results in terms of reproducing business cycle features.[12] Interestingly, the basic findings are largely unchanged. That is, the relative performance of the models in reproducing business cycle features has little to do with the presence of a structural break or regime switching in the variance. The linear models do better than before in terms of capturing variability of growth in recessions and recoveries, but worse in terms of variability of growth in expansions. They also produce shorter expansions than before. The nonlinear models perform very similarly to before, whether or not switching in variance is allowed for. The two-regime model actually does a bit worse, reflecting the fact that a more general model will not necessarily be better at reproducing business cycle features, even if it fits the data better in other dimensions.

<Table 8 Here>

5. Conclusion

It is clear from the analysis in this paper that it is essential in comparing linear and nonlinear models to consider implications of the models that differ in substantive ways. The dimension over which nonlinear models appear to be quite different than linear models is in their ability to reproduce certain features related to business cycle phases, such as a high-growth recovery phase after recessions and a strong correlation between the severity of a recession and the strength of the subsequent recovery. In terms of these features, it is clear that certain nonlinear models vastly outperform linear models. That is, many sample features are inconsistent with linear models, while they can only be reconciled with models that capture certain types of nonlinearities. Thus, combined with recent strong support for nonlinearity using statistical tests, the results presented in this paper suggest that nonlinearity is very relevant to modeling U.S. real GDP as a time-series process.

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[12] To conserve space, we do not report parameter estimates, which are generally similar to those in Tables 3 and 4, but reflect a reduction in volatility since 1984.

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Table 1: Peak and Trough Dates from NBER Business Cycle Dating Committee and the BBQ and MBBQ Algorithms Applied to U.S. Real GDP (1948:Q4 – 2003:Q2)

Business Cycle Peaks			Business Cycle Troughs		
NBER	*BBQ*	*MBBQ*	*NBER*	*BBQ*	*MBBQ*
1948:Q4	**-**	**-**	1949:Q4	**1949:Q2**	1949:Q4
1953:Q2	1953:Q2	1953:Q2	1954:Q2	**1954:Q1**	1954:Q2
1957:Q3	1957:Q3	1957:Q3	1958:Q2	**1958:Q1**	**1958:Q1**
1960:Q2	**1960:Q1**	**1960:Q1**	1961:Q1	**1960:Q4**	**1960:Q4**
1969:Q4	**1969:Q3**	**1969:Q3**	1970:Q4	1970:Q4	1970:Q4
1973:Q4	1973:Q4	1973:Q4	1975:Q1	1975:Q1	1975:Q1
1980:Q1	1980:Q1	1980:Q1	1980:Q3	1980:Q3	1980:Q3
1981:Q3	1981:Q3	1981:Q3	1982:Q4	**1982:Q3**	1982:Q4
1990:Q3	**1990:Q2**	1990:Q3	1991:Q1	1991:Q1	1991:Q1
2001:Q1	**2000:Q4**	2001:Q1	2001:Q4	**2001:Q3**	**2001:Q3**

Notes: Bold denotes identified turning points that differ from the NBER dates. We ignore the first NBER peak date in our evaluation of the BBQ and MBBQ algorithms because, given our sample period, the earliest date at which the algorithms can identify a turning point is 1949:Q2.

Table 2: Business Cycle Features for U.S. Real GDP (1948:Q4 – 2003:Q2)

	NBER	*BBQ*	*MBBQ*
Avg. Quarterly Growth Rates			
Recession	-2.00	-2.52	-2.44
Expansion	4.56	4.58	4.61
Recovery	7.04	5.63	6.67
Mature Expansion	3.93	4.31	4.08
Std. Dev. of Quarterly Growth Rates			
Recession	3.34	2.99	3.05
Expansion	3.54	3.50	3.48
Recovery	4.22	4.50	4.30
Mature Expansion	3.05	3.17	3.04
Number of Phases			
Number of Peaks	9	9	9
Avg. Length of Phases			
Recession	3.44	3.33	3.33
Expansion	19.67	19.89	19.67
Std. Dev. of Length of Phases			
Recession	1.13	1.12	1.41
Expansion	12.72	12.36	12.72
Correlation between Growth Rates			
Recession / Recovery	-0.61	-0.74	-0.75

Note: Because the earliest date at which the algorithms can identify a turning point is 1949:Q2, we ignore the first peak in 1948:Q4 when calculating the sample features associated with the NBER dates.

Table 3: Maximum Likelihood Estimates for Linear Models

Parameter	ARMA(1,0)	ARMA(2,0)	ARMA(0,1)	ARMA(0,2)	ARMA(1,1)	ARMA(2,2)
μ	0.842 (0.099)	0.842 (0.108)	0.838 (0.084)	0.835 (0.097)	0.846 (0.107)	0.852 (0.078)
ϕ_1	0.343 (0.064)	0.314 (0.067)			0.495 (0.125)	1.390 (0.099)
ϕ_2		0.086 (0.068)				-0.759 (0.111)
θ_1			0.270 (0.056)	0.300 (0.067)	-0.174 (0.139)	-1.142 (0.141)
θ_2				0.192 (0.065)		0.597 (0.146)
σ	0.962 (0.046)	0.959 (0.046)	0.977 (0.047)	0.958 (0.046)	0.959 (0.046)	0.932 (0.045)
Log Likelihood	-300.930	-300.119	-304.273	-300.154	-300.231	-294.740
Schwarz Criterion	-308.979	-310.851	-312.322	-310.886	-313.646	-310.838

Note: Standard errors are reported in parentheses.

Table 4: Maximum Likelihood Estimates for Regime-Switching Models

Parameter	Two Regime	Three Regime	Bounceback
p_{11}	0.918 (0.033)	0.917 (0.030)	0.949 (0.022)
p_{22}	0.760 (0.091)	0.723 (0.087)	0.725 (0.100)
p_{33}		0.679 (0.093)	
μ_1	1.166 (0.099)	0.846 (0.090)	0.843 (0.074)
μ_2	-1.312 (0.211)	-1.155 (0.171)	-1.291 (0.271)
μ_3		1.237 (0.182)	
λ			0.309 (0.058)
σ	0.854 (0.048)	0.733 (0.047)	0.788 (0.043)
Log Likelihood	-305.577	-295.135	-292.083
Schwarz Criterion	-318.992	-313.916	-308.181

Note: Standard errors are reported in parentheses.

Table 5: Percentiles of Business Cycle Features for Linear Models

Feature	ARMA(1,0)	ARMA(2,0)	ARMA(0,1)	ARMA(0,2)	ARMA(1,1)	ARMA(2,2)
Avg. Quarterly Growth Rates						
Recession	0.34 (-0.19)	0.30 (-0.23)	0.31 (-0.22)	0.31 (-0.22)	0.30 (-0.23)	0.28 (-0.27)
Expansion	0.77 (+0.27)	0.76 (+0.28)	0.89 (+0.41)	0.82 (+0.32)	0.77 (+0.28)	0.82 (+0.31)
Recovery	**1.00 (+2.31)**	**1.00 (+2.47)**	**1.00 (+2.24)**	**1.00 (+2.30)**	**1.00 (+2.45)**	**0.99 (+1.88)**
Mature Expansion	0.27 (-0.25)	0.27 (-0.28)	0.43 (-0.07)	0.33 (-0.18)	0.28 (-0.27)	0.38 (-0.10)
Std. Dev. of Quarterly Growth Rates						
Recession	**0.96 (+0.79)**	**0.96 (+0.76)**	**0.97 (+0.84)**	**0.97 (+0.80)**	**0.96 (+0.77)**	**0.98 (+0.86)**
Expansion	0.18 (-0.21)	0.19 (-0.20)	0.13 (-0.25)	0.19 (-0.19)	0.19 (-0.20)	0.22 (-0.17)
Recovery	**0.96 (+0.92)**	**0.96 (+0.97)**	**0.93 (+0.81)**	**0.96 (+0.92)**	**0.96 (+0.95)**	**0.95 (+0.86)**
Mature Expansion	**0.00 (-0.70)**	**0.00 (-0.70)**	**0.00 (-0.72)**	**0.00 (-0.69)**	**0.00 (-0.70)**	**0.00 (-0.63)**
Number of Phases						
Number of Peaks	0.44 (0)	0.52 (+1)	0.46 (0)	0.48 (0)	0.52 (+1)	0.54 (+1)
Avg. Length of Phases						
Recession	0.62 (+0.20)	0.47 (-0.03)	0.82 (+1.15)	0.60 (+0.16)	0.53 (+0.06)	0.68 (+0.23)
Expansion	0.51 (+0.11)	0.44 (-0.83)	0.45 (-0.58)	0.47 (-0.33)	0.44 (-0.89)	0.40 (-1.46)
Std. Dev. of Length of Phases						
Recession	0.50 (+0.00)	0.41 (-0.18)	0.67 (+0.29)	0.51 (+0.03)	0.44 (-0.10)	0.67 (+0.22)
Expansion	0.34 (-2.28)	0.29 (-3.22)	0.31 (-2.86)	0.32 (-2.76)	0.30 (-3.21)	0.30 (-3.05)
Correlation between Growth Rates						
Recession / Recovery	**0.02 (-0.80)**	**0.02 (-0.82)**	**0.02 (-0.80)**	**0.03 (-0.79)**	**0.02 (-0.82)**	**0.04 (-0.67)**

Notes: Percentiles are based on 10,000 simulations. They represent the proportion of simulated features that fall below the corresponding sample feature reported in Table 2. Bold denotes a percentile that is less than 0.10 or greater than 0.90. The numbers in parentheses correspond to the difference between a sample feature and the corresponding median simulated feature.

Table 6: Percentiles of Business Cycle Features for Regime-Switching Models

Feature	Two Regime	Three Regime	Bounceback
Avg. Quarterly Growth Rates			
Recession	0.36 (-0.16)	0.33 (-0.18)	0.55 (+0.07)
Expansion	0.79 (+0.25)	0.79 (+0.31)	0.85 (+0.35)
Recovery	**1.00 (+2.41)**	0.85 (+0.99)	0.67 (+0.46)
Mature expansion	0.21 (-0.30)	0.59 (+0.09)	0.79 (+0.23)
Std. Dev. of Quarterly Growth Rates			
Recession	0.88 (+0.52)	**0.98 (+0.82)**	0.90 (+0.59)
Expansion	0.24 (-0.14)	0.18 (-0.23)	0.25 (-0.17)
Recovery	**0.95 (+0.86)**	0.74 (+0.33)	0.63 (+0.20)
Mature Expansion	**0.00 (-0.61)**	**0.04 (-0.52)**	**0.07 (-0.34)**
Number of Phases			
Number of Peaks	0.61 (+1)	0.65 (+1)	0.68 (+1)
Avg. Length of Phases			
Recession	0.31 (-0.38)	0.35 (-0.30)	0.35 (-0.34)
Expansion	0.40 (-1.71)	0.35 (-2.47)	0.32 (-2.95)
Std. Dev. of Length of Phases			
Recession	026 (-0.56)	0.28 (-0.50)	0.28 (-0.53)
Expansion	0.24 (-4.25)	0.27 (-3.96)	0.25 (-4.43)
Correlation between Growth Rates			
Recession / Recovery	**0.03 (-0.82)**	**0.06 (-0.59)**	0.18 (-0.27)

Notes: Percentiles are based on 10,000 simulations. They represent the proportion of simulated features that fall below the corresponding sample feature reported in Table 2. Bold denotes a percentile that is less than 0.10 or greater than 0.90. The numbers in parentheses correspond to the difference between a sample feature and the corresponding median simulated feature.

Table 7: Percentiles of Business Cycle Features for Linear Models with a Structural Break

Feature	ARMA(1,0)	ARMA(2,0)	ARMA(0,1)	ARMA(0,2)	ARMA(1,1)	ARMA(2,2)
Avg. Quarterly Growth Rates						
Recession	0.65 (-0.20)	0.62 (+0.17)	0.61 (+0.15)	0.62 (+0.17)	0.61 (+0.16)	0.59 (+0.11)
Expansion	0.61 (+0.14)	0.59 (+0.12)	0.73 (+0.28)	0.62 (+0.15)	0.60 (+0.15)	0.60 (+0.12)
Recovery	**0.98 (+1.95)**	**0.98 (+2.20)**	**0.98 (+1.95)**	**0.97 (+1.95)**	**0.98 (+2.47)**	**0.96 (+1.80)**
Mature Expansion	0.30 (-0.30)	0.26 (-0.41)	0.41 (-0.12)	0.32 (-0.28)	0.27 (-0.37)	0.31 (-0.28)
Std. Dev. of Quarterly Growth Rates						
Recession	0.72 (+0.31)	0.67 (+0.23)	0.74 (+0.35)	0.72 (+0.30)	0.69 (+0.27)	0.76 (+0.36)
Expansion	**0.09 (-0.53)**	**0.08 (-0.56)**	**0.05 (-0.60)**	**0.08 (-0.56)**	**0.09 (-0.53)**	0.10 (-0.50)
Recovery	0.76 (+0.43)	0.78 (+0.51)	0.69 (+0.30)	0.75 (+0.43)	0.78 (+0.51)	0.77 (+0.47)
Mature Expansion	**0.01 (-0.99)**	**0.01 (-1.05)**	**0.01 (-1.04)**	**0.01 (-1.02)**	**0.01 (-1.20)**	**0.01 (-0.95)**
Number of Phases						
Number of Peaks	0.60 (+1)	0.72 (+2)	0.63 (+1)	0.66 (+1)	0.70 (+2)	0.72 (+2)
Avg. Length of Phases						
Recession	0.45 (-0.07)	0.27 (-0.50)	0.66 (+0.33)	0.42 (-0.11)	0.32 (-0.34)	0.36 (-0.22)
Expansion	0.70 (+3.24)	0.64 (+2.38)	0.68 (+3.07)	0.67 (+3.00)	0.65 (+2.56)	0.63 (+2.10)
Std. Dev. of Length of Phases						
Recession	0.37 (-0.24)	0.26 (-0.58)	0.53 (+0.06)	0.39 (-0.19)	0.29 (-0.48)	0.40 (-0.16)
Expansion	0.48 (-0.35)	0.44 (-1.05)	0.48 (-0.31)	0.48 (-0.38)	0.45 (-0.86)	0.47 (-0.55)
Correlation between Growth Rates						
Recession / Recovery	**0.03 (-0.75)**	**0.04 (-0.77)**	**0.03 (-0.75)**	**0.04 (-0.74)**	**0.04 (-0.78)**	**0.05 (-0.65)**

Notes: Percentiles are based on 10,000 simulations. They represent the proportion of simulated features that fall below the corresponding sample feature reported in Table 2. Bold denotes a percentile that is less than 0.10 or greater than 0.90. The numbers in parentheses correspond to the difference between a sample feature and the corresponding median simulated feature. The structural break occurs in 1984:Q1.

Table 8: Percentiles of Business Cycle Features for Regime-Switching Models with a Structural Break

Feature	Two Regime	Two Regime Switching Variance	Three Regime	Three Regime Switching Variance	Bounceback	Bounceback Switching Variance
Avg. Quarterly Growth Rates						
Recession	0.42 (-0.13)	0.36 (-0.19)	0.26 (-0.32)	0.29 (-0.30)	0.49 (-0.01)	0.71 (+0.33)
Expansion	**0.93 (+0.55)**	**0.92 (+0.54)**	**0.92 (+0.50)**	0.83 (+0.34)	0.90 (+0.50)	0.90 (+0.50)
Recovery	**0.99 (+2.31)**	**0.99 (+2.29)**	**0.97 (+2.02)**	**0.93 (+1.46)**	0.78 (+0.87)	0.80 (+0.97)
Mature Expansion	0.60 (+0.09)	0.59 (+0.09)	0.59 (+0.08)	0.53 (+0.03)	0.85 (+0.35)	0.83 (+0.32)
Std. Dev. of Quarterly Growth Rates						
Recession	0.69 (+0.31)	0.75 (+0.40)	0.84 (+0.55)	0.82 (+0.52)	0.83 (+0.52)	0.53 (+0.05)
Expansion	**0.08 (-0.46)**	**0.07 (-0.48)**	0.20 (-0.24)	0.11 (-0.37)	0.13 (-0.36)	0.13 (-0.36)
Recovery	0.69 (+0.31)	0.68 (+0.30)	0.79 (+0.49)	0.56 (+0.09)	0.43 (-0.12)	0.46 (-0.08)
Mature Expansion	**0.01 (-0.88)**	**0.01 (-0.90)**	**0.02 (-0.63)**	**0.03 (-0.67)**	**0.04 (-0.52)**	**0.04 (-0.55)**
Number of Phases						
Number of Peaks	0.69 (+2)	0.69 (+2)	0.67 (+1)	0.68 (+1)	0.70 (+2)	0.72 (+2)
Avg. Length of Phases						
Recession	0.50 (+0.00)	0.47 (-0.00)	0.36 (-0.27)	0.32 (-0.37)	0.37 (-0.27)	0.45 (-0.10)
Expansion	0.44 (-0.96)	0.46 (-0.73)	0.41 (-1.43)	0.38 (-2.13)	0.41 (-1.63)	0.37 (-2.23)
Std. Dev. of Length of Phases						
Recession	0.41 (-0.19)	0.39 (-0.23)	0.30 (-0.47)	0.26 (-0.59)	0.30 (-0.51)	0.36 (-0.32)
Expansion	0.30 (-3.75)	0.30 (-3.47)	0.29 (-3.51)	0.27 (-3.94)	0.30 (-3.49)	0.27 (-4.25)
Correlation between Growth Rates						
Recession / Recovery	**0.04 (-0.78)**	**0.04 (-0.78)**	**0.04 (-0.79)**	**0.05 (-0.80)**	0.19 (-0.27)	0.20 (-0.27)

Notes: Percentiles are based on 10,000 simulations. They represent the proportion of simulated features that fall below the corresponding sample feature reported in Table 2. Bold denotes a percentile that is less than 0.10 or greater than 0.90. The numbers in parentheses correspond to the difference between a sample feature and the corresponding median simulated feature. The structural break occurs in 1984:Q1.