Econometrics: Models of Regime Changes

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Glossary

Filtered Probability of a Regime

The probability that the unobserved Markov chain for a Markov-switching model is in a particular regime in period *t*, conditional on observing sample information up to period *t*.

Gibbs Sampler

An algorithm to generate a sequence of samples from the joint probability distribution of a group of random variables by repeatedly sampling from the full set of conditional distributions for the random variables.

Markov Chain

A process that consists of a finite number of states, or regimes, where the probability of moving to a future state conditional on the present state is independent of past states.

Markov-Switching Model

A regime-switching model in which the shifts between regimes evolve according to an unobserved Markov chain.

Regime-Switching Model

A parametric model of a time series in which parameters are allowed to take on different values in each of some fixed number of regimes.

Smooth Transition Threshold Model

A threshold model in which the effect of a regime shift on model parameters is phased in gradually, rather than occurring abruptly.

Smoothed Probability of a Regime

The probability that the unobserved Markov chain for a Markov-switching model is in a particular regime in period *t*, conditional on observing all sample information.

Threshold Model

A regime-switching model in which the shifts between regimes are triggered by the level of an observed economic variable in relation to an unobserved threshold.

Time-Varying Transition Probability

A transition probability for a Markov chain that is allowed to vary depending on the outcome of observed information.

Transition Probability

The probability that a Markov chain will move from state *j* to state *i*.

I. Definition of the Subject and Its Importance

Regime-switching models are time-series models in which parameters are allowed to take on different values in each of some fixed number of "regimes." A stochastic process assumed to have generated the regime shifts is included as part of the model, which allows for model-based forecasts that incorporate the possibility of future regime shifts. In certain special situations the regime in operation at any point in time is directly observable. More generally the regime is unobserved, and the researcher must conduct inference about which regime the process was in at past points in time. The primary use of these models in the applied econometrics literature has been to describe changes in the dynamic behavior of macroeconomic and financial time series.

Regime-switching models can be usefully divided into two categories, "threshold" models and "Markov-switching" models. The primary difference between these approaches is in how the evolution of the state process is modeled. Threshold models, introduced by Tong (1983), assume that regime shifts are triggered by the level of observed variables in relation to an unobserved threshold. Markov-switching models, introduced to econometrics by Goldfeld and Quandt (1973), Cosslett and Lee (1985), and Hamilton (1989), assume that the regime shifts evolve according to a Markov chain.

Regime-switching models have become an enormously popular modeling tool for applied work. Of particular note are regime-switching models of measures of economic output, such as real Gross Domestic Product (GDP), which have been used to model and identify the phases of the business cycle. Examples of such models include Hamilton (1989), Beaudry and Koop (1993), Tiao and Tsay (1994), Potter (1995), Pesaran and Potter (1997), Chauvet (1998), Van Dijk and Franses (1999), Kim and

Nelson (1999b, 1999c), Öcal and Osborne (2000), and Kim, Morley and Piger (2005). A sampling of other applications include modeling regime shifts in time-series of inflation and interest rates (Evans and Wachtel, 1993; Garcia and Perron, 1996; Ang and Bekaert, 2002), high and low volatility regimes in equity returns (Turner, Startz and Nelson, 1989; Hamilton and Susmel, 1994; Hamilton and Lin, 1996; Dueker, 1997), shifts in the Federal Reserve's policy "rule" (Kim, 2004; Sims and Zha, 2006), and time variation in the response of economic output to monetary policy actions (Garcia and Schaller, 2002; Kaufmann, 2002; Ravn and Sola, 2004; Lo and Piger, 2005).

II. Introduction

There is substantial interest in modeling the dynamic behavior of macroeconomic and financial quantities observed over time. A challenge for this analysis is that these time series likely undergo changes in their behavior over reasonably long sample periods. This change may occur in the form of a "structural break", in which there is a shift in the behavior of the time series due to some permanent change in the economy's structure. Alternatively, the change in behavior might be temporary, as in the case of wars or "pathological" macroeconomic episodes such as economic depressions, hyperinflations, or financial crises. Finally, such shifts might be both temporary and recurrent, in that the behavior of the time series might cycle between regimes. For example, early students of the business cycle argued that the behavior of economic variables changed dramatically in business cycle expansions vs. recessions.

The potential for shifts in the behavior of economic time series means that constant parameter time series models might be inadequate for describing their evolution.

As a result, recent decades have seen extensive interest in econometric models designed to incorporate parameter variation. One approach to describing this variation, denoted a "regime-switching" model in the following, is to allow the parameters of the model to take on different values in each of some fixed number of regimes, where, in general, the regime in operation at any point in time is unobserved by the econometrician. However, the process that determines the arrival of new regimes is assumed known, and is incorporated into the stochastic structure of the model. This allows the econometrician to draw inference about the regime that is in operation at any point in time, as well as form forecasts of which regimes are most likely in the future.

Application of regime-switching models are usually motivated by economic phenomena that appear to involve cycling between recurrent regimes. For example, regime-switching models have been used to investigate the cycling of the economy between business cycle phases (expansion and recession), "bull" and "bear" markets in equity returns, and high and low volatility regimes in asset prices. However, regime switching models need not be restricted to parameter movement across recurrent regimes. In particular, the regimes might be non-recurrent, in which case the models can capture permanent "structural breaks" in model parameters.

There are a number of formulations of regime-switching time-series models in the recent literature, which can be usefully divided into two broad approaches. The first models regime change as arising from the observed behavior of the level of an economic variable in relation to some threshold value. These "threshold" models were first introduced by Tong (1983), and are surveyed by Potter (1999). The second models regime change as arising from the outcome of an unobserved, discrete, random variable, which is assumed to follow a Markov process. These models, commonly referred to as "Markov-switching" models, were introduced in econometrics by Goldfeld and Quandt (1973) and Cosslett and Lee (1985), and became popular for applied work following the seminal contribution of Hamilton (1989). Hamilton and Raj (2002) and Hamilton (2005a) provide surveys of Markov-switching models, while Hamilton (1994) and Kim and Nelson (1999a) provide textbook treatments.

There are by now a number of empirical applications of regime-switching models that establish their empirical relevance over constant parameter alternatives. In particular, a large literature has evaluated the statistical significance of regime-switching autoregressive models of measures of U.S. economic activity. While the early literature did not find strong evidence for simple regime-switching models over the alternative of a constant parameter autoregression for U.S. real GDP (e.g. Garcia, 1998), later researchers have found stronger evidence using more complicated models of real GDP (Kim, Morley and Piger, 2005), alternative measures of economic activity (Hamilton, 2005b), and multivariate techniques (Kim and Nelson, 2001). Examples of other studies finding statistical evidence in favor of regime-switching models include Garcia and Perron (1996), who document regime switching in the conditional mean of an autoregression for the U.S. real interest rate, and Guidolin and Timmermann (2005), who find evidence of regime-switching in the conditional mean and volatility of U.K. equity returns.

This article surveys the literature surrounding regime-switching models, focusing primarily on Markov-switching models. The organization of the article is as follows. Section III describes both threshold and Markov-switching models using a simple example. The article then focuses on Markov-switching models, with Section IV discussing estimation techniques for a basic model, Section V surveying a number of primary extensions of the basic model, and Section VI surveying issues related to specification analysis. Section VII gives an empirical example, discussing how Markov-switching models can be used to identify turning points in the U.S. business cycle. The article concludes by highlighting some particular avenues for future research.

III. Threshold and Markov-Switching Models of Regime Change

This section describes the threshold and Markov-switching approaches to modeling regime-switching using a specific example. In particular, suppose we are interested in modeling the sample path of a time series, $\{y_t\}_{t=1}^{T}$, where y_t is a scalar, stationary, random variable. A popular choice is an autoregressive (AR) model of order *k*:

$$y_t = \alpha + \sum_{j=1}^{k} \phi_j y_{t-j} + \varepsilon_t \,, \tag{1}$$

where the disturbance term, ε_t, is assumed to be normally distributed, so that $\varepsilon_t \sim N(0, \sigma^2)$. The AR(*k*) model in (1) is a parsimonious description of the data, and has a long history as a tool for establishing stylized facts about the dynamic behavior of the time series, as well as an impressive record in forecasting.

In many cases however, we might be interested in whether the behavior of the time series changes across different periods of time, or regimes. In particular, we may be interested in the following regime-switching version of (1):

$$y_t = \alpha_{S_t} + \sum_{j=1}^{k} \phi_{j,S_t} y_{t-j} + \varepsilon_t \ , \tag{2}$$

where $\varepsilon_t \sim N\left(0, \sigma_{S_t}^2\right)$. In (2), the parameters of the AR(k) depend on the value of a discrete-valued state variable, $S_t = i$, i = 1,….,N, which denotes the regime in operation at time t. Put simply, the parameters of the AR(k) model are allowed to vary among one of N different values over the sample period.

There are several items worth emphasizing about the model in (2). First, conditional on being inside of any particular regime, eq. (2) is simply a constant parameter linear regression. Such models, which are commonly referred to as "piecewise linear", make up the vast majority of the applications of regime-switching models. Second, if the state variable were observed, the model in (2) is simply a linear regression model with dummy variables, a fact that will prove important in our discussion of how the parameters of (2) might be estimated. Third, although the specification in (2) allows for all parameters to switch across all regimes, more restrictive models are certainly possible, and indeed are common in applied work. For example, a popular model for time series of asset prices is one in which only the variance of the disturbance term is allowed to vary across regimes. Finally, the shifts in the parameters of (2) are modeled as occurring abruptly. An example of an alternative approach, in which parameter shifts are phased in gradually, can be found in the literature investigating "smooth transition" threshold models. Such models will not be described further here, but are discussed in detail in Granger and Teräsvirta (1993).

Threshold and Markov-switching models differ in the assumptions made about the state variable, S_t. Threshold models assume that S_t is a deterministic function of an observed variable. In most applications this variable is taken to be a particular lagged value of the process itself, in which case regime shifts are said to be "self-exciting". In particular, define *N-1* "thresholds" as $\tau_1 < \tau_2 < .... < \tau_{N-1}$. Then, for a self-exciting threshold model, S_t is defined as follows:

$$
\begin{array}{ll}
S_t = 1 & y_{t-d} < \tau_1, \\
S_t = 2 & \tau_1 \le y_{t-d} < \tau_2 \\
. & . \qquad\qquad . \\
. & . \\
S_t = N & \tau_{N-1} \le y_{t-d}
\end{array}
\tag{3}
$$

In (3), *d* is known as the "delay" parameter. In most cases S_t is unobserved by the econometrician, because the delay and thresholds, *d* and τ_i, are generally not observable. However, *d* and τ_i can be estimated along with other model parameters. Potter (1999) surveys classical and Bayesian approaches to estimation of the parameters of threshold models.

Markov-switching models also assume that S_t is unobserved. In contrast to threshold models however, S_t is assumed to follow a particular stochastic process, namely an *N*-state Markov chain. The evolution of Markov chains are described by their transition probabilities, given by:

$$P(S_t = i \mid S_{t-1} = j, S_{t-2} = q, ...) = P(S_t = i \mid S_{t-1} = j) = p_{ij}, \tag{4}$$

where, conditional on a value of *j,* we assume $\sum_{i=1}^{N} p_{ij} = 1$. That is, the process in (4) specifies a complete probability distribution for S_t. In the general case, the Markov process allows regimes to be visited in any order and for regimes to be visited more than once. However, restrictions can be placed on the p_{ij} to restrict the order of regime shifts. For example, Chib (1998) notes that the transition probabilities can be restricted in such a way so that the model in (2) becomes a "changepoint" model in which there are *N*-1 structural breaks in the model parameters. Finally, the vast majority of the applied literature has assumed that the transition probabilities in (4) evolve independently of lagged values of the series itself, so that

$$P\left(S_t = i \mid S_{t-1} = j, S_{t-2} = q, \ldots, y_{t-1}, y_{t-2}, \ldots\right) = P\left(S_t = i \mid S_{t-1} = j\right) = p_{ij}, \qquad (5)$$

which is the polar opposite of the threshold process described in (3). For this reason, Markov-switching models are often described as having regimes that evolve "exogenously" of the series, while threshold models are said to have "endogenous" regimes. However, while popular in practice, the restriction in (5) is not necessary for estimation of the parameters of the Markov-switching model. Section V of this article discusses models in which the transition probabilities of the Markov process are allowed to be partially determined by lagged values of the series.

The threshold and Markov-switching approaches are best viewed as complementary, with the "best" model likely to be application specific. Certain applications appear tailor-made for the threshold assumption. For example, we might have good reason to think that the behavior of time series such as an exchange rate or inflation will exhibit regime shifts when the series moves outside of certain thresholds, as this will trigger government intervention. The Markov-switching model might instead be the obvious choice when one does not wish to tie the regime shifts to the behavior of a particular observed variable, but instead wishes to let the data speak freely as to when regime shifts have occurred.

In the remainder of this article I will survey various aspects regarding the econometrics of Markov-switching models. For readers interested in learning more about threshold models, the survey article of Potter (1999) is an excellent starting point.

IV. Estimation of a Basic Markov-Switching Model

This section discusses estimation of the parameters of Markov-switching models. The existing literature has focused almost exclusively on likelihood based methods for estimation. I retain this focus here, and discuss both maximum likelihood and Bayesian approaches to estimation. An alternative approach based on semi-parametric estimation is discussed in Campbell (2002).

To aid understanding, we focus on a specific, baseline, case, which is the Markov-switching autoregression given in (2) and (5). We simplify further by allowing for $N = 2$ regimes, so that $S_t = 1$ or 2. It is worth noting that in many cases two regimes is a reasonable assumption. For example, in the literature using Markov-switching models to study business cycles phases, a two regime model, meant to capture an expansion and recession phase, is an obvious starting point that has been used extensively.

Estimation of Markov-switching models necessitates two additional restrictions over constant parameter models. First of all, the labeling of S_t is arbitrary, in that switching the vector of parameters associated with $S_t = 1$ and $S_t = 2$ will yield an identical model. A commonly used approach to normalize the model is to restrict the value of one of the parameters when $S_t = 1$ relative to its value when $S_t = 2$. For example, for the model in (2) we could restrict $\alpha_2 < \alpha_1$. For further details on the choice of normalization, see Hamilton, Wagoner and Zha (2004). Second, the transition probabilities in (5) must be constrained to lie in [0,1]. One approach to implement this constraint, which will be useful in later discussion, is to use a probit specification for S_t. In particular, the value of S_t is assumed to be determined by the realization of a random variable, η_t, as follows:

$$S_t = \begin{cases} 1 & \text{if} \quad \eta_t < \gamma_{S_{t-1}} \\ 2 & \text{if} \quad \eta_t \geq \gamma_{S_{t-1}} \end{cases}, \tag{6}$$

where $\eta_t \sim i.i.d.N(0,1)$. The specification in (6) depends on two parameters, γ_1 and γ_2, which determine the transition probabilities of the Markov process as follows:

$$\begin{aligned} p_{1j} &= P(\eta_t < \gamma_j) = \Phi(\gamma_j) \\ p_{2j} &= 1 - p_{1j} \end{aligned}, \tag{7}$$

where j = 1, 2 and Φ is the standard normal cumulative distribution function.

There are two main items of interest on which to conduct statistical inference for Markov-switching models. The first is the parameters of the model, of which there are $2(k+3)$ for the two-regime Markov-switching autoregression. In the following we collect these parameters in the vector

$$\theta = \left(\alpha_1, \phi_{1,1}, \phi_{2,1}, \ldots, \phi_{k,1}, \sigma_1, \alpha_2, \phi_{1,2}, \phi_{2,2}, \ldots, \phi_{k,2}, \gamma_1, \gamma_2\right)'. \tag{8}$$

The second item of interest is the regime indicator variable, S_t. In particular, as S_t is unobserved, we will be interested in constructing estimates of which regime was in operation at each point in time. These estimates will take the form of posterior probabilities that $S_t = i$, i =1, 2. We assume that the econometrician has a sample of T + k observations, $\left(y_T, y_{T-1}, y_{T-2}, \ldots\ldots, y_{-(k-1)}\right)$. The series of observations available up to time t is denoted as $\Omega_t = \left(y_t, y_{t-1}, y_{t-2}, \ldots\ldots, y_{-(k-1)}\right)$.

We begin with maximum likelihood estimation of θ. Maximum likelihood estimation techniques for various versions of Markov-switching regressions can be found in the existing literature of multiple disciplines, for example Poritz (1982), Juang and Rabiner (1985), and Rabiner (1989) in the speech recognition literature, and Cosslett and Lee (1985) and Hamilton (1989) in the econometrics literature. Here we focus on the presentation of the problem given in Hamilton (1989), who presents a simple iterative algorithm that can be used to construct the likelihood function of a Markov-switching autoregression, as well as compute posterior probabilities for S_t.

For a given value of θ, the conditional log likelihood function is given by:

$$L(\theta) = \sum_{t=1}^{T} \log f\left(y_t \mid \Omega_{t-1}; \theta\right). \tag{9}$$

Construction of the conditional log likelihood function then requires construction of the conditional density function, $f\left(y_t \mid \Omega_{t-1}; \theta\right)$, for t = 1,…,T. The "Hamilton Filter" computes these conditional densities recursively as follows: Suppose for the moment that we are given $P(S_{t-1} = j \mid \Omega_{t-1};\theta)$, which is the posterior probability that $S_{t-1} = j$ based on information observed through period *t-1*. Equations (10) and (11) can then be used to construct $f(y_t \mid \Omega_{t-1};\theta)$:

$$P(S_t = i \mid \Omega_{t-1};\theta) = \sum_{j=1}^{2} P(S_t = i \mid S_{t-1} = j, \Omega_{t-1};\theta) P(S_{t-1} = j \mid \Omega_{t-1};\theta), \qquad (10)$$

$$f(y_t \mid \Omega_{t-1};\theta) = \sum_{i=1}^{2} f(y_t \mid S_t = i, \Omega_{t-1};\theta) P(S_t = i \mid \Omega_{t-1};\theta). \qquad (11)$$

From eq. (5), the first term in the summation in (10) is simply the transition probability, p_{ij}, which is known for any particular value of θ. The first term in (11) is the conditional density of y_t assuming that $S_t = i$, which, given the within-regime normality assumption for ε_t, is:

$$f(y_t \mid S_1 = i, \Omega_0;\theta) = \frac{1}{\sigma_i \sqrt{2\pi}} \exp\left(\frac{-\left(y_t - \alpha_i - \sum_{j=1}^{k} \phi_{j,i} y_{t-j} \right)^2}{2\sigma_i^2} \right). \qquad (12)$$

With $f(y_t \mid \Omega_{t-1};\theta)$ in hand, the next step is then to update (10) and (11) to compute $f(y_{t+1} \mid \Omega_t;\theta)$. To do so requires $P(S_t = i \mid \Omega_t;\theta)$ as an input, meaning we must update $P(S_t = i \mid \Omega_{t-1};\theta)$ to reflect the information contained in y_t. This updating is done using Bayes' rule:

$$P(S_t = i \mid \Omega_t;\theta) = \frac{f(y_t \mid S_t = i, \Omega_{t-1};\theta) P(S_t = i \mid \Omega_{t-1})}{f(y_t \mid \Omega_{t-1};\theta)}, \qquad (13)$$

where each of the three elements on the right-hand side of (13) are computable from the elements of (10) and (11). Given a value for $P(S_0 = i \mid \Omega_0; \theta)$ to initialize the filter, equations (10)-(13) can then be iterated to construct $f(y_t \mid \Omega_{t-1}; \theta)$, $t = 1,\ldots,T,$ and therefore the log likelihood function, $L(\theta)$. The maximum likelihood estimates, $\hat{\theta}_{MLE}$, are then the value of θ that maximizes $L(\theta)$, and can be obtained using standard numerical optimization techniques.

How do we set $P(S_0 = i \mid \Omega_0; \theta)$ to initialize the filter? As is discussed in Hamilton (1989), exact evaluation of this probability is rather involved. The usual practice, which is possible when S_t is an ergodic Markov chain, is to simply set $P(S_0 = i \mid \Omega_0; \theta)$ equal to the unconditional probability, $P(S_0 = i)$. For the two regime case considered here these unconditional probabilities are given by:

$$P(S_0 = 1) = \frac{1 - p_{22}}{2 - p_{11} - p_{22}} \tag{14}$$

$$P(S_0 = 2) = 1 - P(S_0 = 1).$$

Alternatively, $P(S_0 = i \mid \Omega_0; \theta)$ could be treated as an additional parameter to be estimated. See Hamilton (1994) and Kim and Nelson (1999a) for further details.

An appealing feature of the Hamilton filter is that, in addition to the likelihood function, the procedure also directly evaluates $P(S_t = i \mid \Omega_t; \theta)$, which is commonly referred to as a “filtered” probability. Inference regarding the value of S_t is then sometimes based on $P(S_t = i \mid \Omega_t; \hat{\theta}_{MLE})$, which is obtained by running the Hamilton filter with $\theta = \hat{\theta}_{MLE}$. In many circumstances, we might also be interested in the so-called "smoothed" probability of a regime computed using all available data, or $P(S_t = i \mid \Omega_T; \theta)$. Kim (1994) presents an efficient recursive algorithm that can be applied to compute these smoothed probabilities.

We now turn to Bayesian estimation of Markov-switching models. In the Bayesian approach, the parameters θ are themselves assumed to be random variables, and the goal is to construct the posterior density for these parameters given the observed data, denoted $f(\theta \mid \Omega_t)$. In all but the simplest of models, this posterior density does not take the form of any well known density whose properties can be analyzed analytically. In this case, modern Bayesian inference usually proceeds by sampling the posterior density repeatedly to form estimates of posterior moments and other objects of interest. These estimates can be made arbitrarily accurate by increasing the number of samples taken from the posterior. In the case of Markov-switching models, Albert and Chib (1993) demonstrate that samples from $f(\theta \mid \Omega_t)$ can be obtained using a simulation-based approach known as the Gibbs Sampler. The Gibbs Sampler, introduced by Geman and Geman (1984), Tanner and Wong (1987) and Gelfand and Smith (1990), is an algorithm that produces random samples from the joint density of a group of random variables by repeatedly sampling from the full set of conditional densities for the random variables.

We will sketch out the main ideas of the Gibbs Sampler in the context of the two-regime Markov-switching autoregression. It will prove useful to divide the parameter space into $\theta = (\theta_1, \theta_2)'$, where $\theta_1 = (\alpha_1, \phi_{1,1}, \phi_{2,1}, ..., \phi_{k,1}, \sigma_1, \alpha_2, \phi_{1,2}, \phi_{2,2}, ..., \phi_{k,2})'$ and

$\theta_2 = (\gamma_1, \gamma_2)$. Suppose it is feasible to simulate draws from the three conditional distributions, $f(\theta_1 \mid \theta_2, \widetilde{S}, \Omega_T)$, $f(\theta_2 \mid \theta_1, \widetilde{S}, \Omega_T)$, and $P(\widetilde{S} \mid \theta_1, \theta_2, \Omega_T)$, where $\widetilde{S} = (S_1, S_2, ..., S_T)'$. Then, conditional on arbitrary initial values, $\theta_2^{(0)}$ and $\widetilde{S}^{(0)}$, we can obtain a draw of θ_1, denoted $\theta_1^{(1)}$, from $f(\theta_1 \mid \theta_2^{(0)}, \widetilde{S}^{(0)}, \Omega_T)$, a draw of θ_2, denoted $\theta_2^{(1)}$, from $f(\theta_2 \mid \theta_1^{(1)}, \widetilde{S}^{(0)}, \Omega_T)$, and a draw of $\widetilde{S}$, denoted $\widetilde{S}^{(1)}$, from $P(\widetilde{S} \mid \theta_1^{(1)}, \theta_2^{(1)}, \Omega_T)$. This procedure can be iterated to obtain $\theta_1^{(j)}$, $\theta_2^{(j)}$, and $\widetilde{S}^{(j)}$, for j = 1, …, J. For large enough J, and assuming weak regularity conditions, these draws will converge to draws from $f(\theta \mid \Omega_T)$ and $P(\widetilde{S} \mid \Omega_T)$. Then, by taking a large number of such draws beyond J, one can estimate any feature of $f(\theta \mid \Omega_T)$ and $P(\widetilde{S} \mid \Omega_T)$, such as moments of interest, with an arbitrary degree of accuracy. For example, an estimate of $P(S_t = i \mid \Omega_T)$ can be obtained by computing the proportion of draws of $\widetilde{S}$ for which $S_t = i$.

Why is the Gibbs Sampler useful for a Markov-switching model? It turns out that although $f(\theta \mid \Omega_t)$ and $P(\widetilde{S} \mid \Omega_T)$ cannot be sampled directly, it is straightforward, assuming natural conjugate prior distributions, to obtain samples from $f(\theta_1 \mid \theta_2, \widetilde{S}, \Omega_T)$, $f(\theta_2 \mid \theta_1, \widetilde{S}, \Omega_T)$, and $P(\widetilde{S} \mid \theta_1, \theta_2, \Omega_T)$. This is most easily seen for the case of θ_1, which, when $\widetilde{S}$ is conditioning information, represents the parameters of a linear regression with dummy variables, a case for which techniques to sample the parameter posterior distribution are well established (Zellner, 1971). An algorithm for obtaining draws of $\widetilde{S}$ from $P(\widetilde{S} \mid \theta_1, \theta_2, \Omega_T)$ was first given in Albert and Chib (1993), while Kim and Nelson (1998) develop an alternative, efficient, algorithm based on the notion of "multi-move" Gibbs Sampling introduced in Carter and Kohn (1994). For further details regarding the implementation of the Gibbs Sampler in the context of Markov-switching models, see Kim and Nelson (1999a).

The Bayesian approach has a number of features that make it particularly attractive for estimation of Markov-switching models. First of all, the requirement of prior density functions for model parameters, considered by many to be a weakness of the Bayesian approach in general, is often an advantage for Bayesian analysis of Markov-switching models (Hamilton, 1991). For example, priors can be used to push the model toward capturing one type of regime-switching vs. another. The value of this can be seen for Markov-switching models of the business cycle, for which the econometrician might wish to focus on portions of the likelihood surface related to business cycle switching, rather than those related to longer term regime shifts in productivity growth. Another advantage of the Bayesian approach is with regards to the inference drawn on S_t. In the maximum likelihood approach, the methods of Kim (1994) can be applied to obtain $P\left(S_t = i \mid \Omega_T ; \hat{\theta}_{MLE}\right)$. As these probabilities are conditioned on the maximum likelihood parameter estimates, uncertainty regarding the unknown values of the parameters has not been taken into account. By contrast, the Bayesian approach yields $P\left(S_t = i \mid \Omega_T\right)$, which is not conditional on a particular value of θ and thus incorporates uncertainty regarding the value of θ that generated the observed data.

V. Extensions of the Basic Markov-Switching Model

The basic, two-regime Markov-switching autoregression in (2) and (5) has been used extensively in the literature, and remains a popular specification in applied work.

However, it has been extended in a number of directions in the substantial literature that follows Hamilton (1989). This section surveys a number of these extensions.

The estimation techniques discussed in section IV can be adapted in a straightforward manner to include several extensions to the basic Markov-switching model. For example, the filter in (10)-(13) can be modified in obvious ways to incorporate the case of $N > 2$ regimes, as well as to allow y_t to be a vector of random variables, so that the model in (2) becomes a Markov-switching vector autoregression (MS-VAR). Hamilton (1994) discusses both of these cases, while Krolzig (1997) provides extensive discussion of MS-VARs. Sims and Zha (2006) is a recent example of applied work using a MS-VAR with a large number of regimes. In addition, the (known) within-regime distribution of the disturbance term, ε_t, could be non-Gaussian, as in Dueker (1997) or Hamilton (2005b). Further, the parameters of (2) could be extended to depend not just on S_t, but also on a finite number of lagged values of S_t, or even a second state variable possibly correlated with S_t. Indeed, such processes can generally be rewritten in terms of the current value of a single, suitably redefined, state variable. Kim and Murray (2002) and Kim, Piger and Startz (2007) provide examples of such a redefinition. For further discussion of all of these cases, see Hamilton (1994).

The specification for the transition probabilities in (5) restricted the probability that $S_t = i$ to depend only on the value of S_{t-1}. However, in some applications we might think that these transition probabilities are driven in part by observed variables, such as the past evolution of the process. To this end, Diebold, Lee and Weinbach (1994) and Filardo (1994) develop Markov-switching models with time-varying transition probabilities (TVTP), in which the transition probabilities are allowed to vary depending on conditioning information. Suppose that z_t represents a vector of observed variables that are thought to influence the realization of the regime. The probit representation for the state process in (6) and (7) can then be extended as follows:

$$S_t = \begin{cases} 1 \text{ if } \eta_t < \left(\gamma_{S_{t-1}} + z_t^{'}\lambda_{S_{t-1}}\right) \\ 2 \text{ if } \eta_t \geq \left(\gamma_{S_{t-1}} + z_t^{'}\lambda_{S_{t-1}}\right) \end{cases}, \tag{15}$$

with associated transition probabilities:

$$\begin{aligned} p_{1j}(z_t) &= P\left(\eta_t < \left(\gamma_j + z_t^{'}\lambda_j\right)\right) = \Phi\left(\gamma_j + z_t^{'}\lambda_j\right) \\ p_{2j}(z_t) &= 1 - p_{1j}(z_t) \end{aligned}, \tag{16}$$

where j = 1, 2 and Φ is again the standard normal cumulative distribution function. Estimation of the Markov-switching autoregression with TVTP is then straightforward. In particular, assuming that z_t contains lagged values of y_t or exogenous random variables, maximum likelihood estimation proceeds by simply replacing p_{ij} with $p_{ij}(z_t)$ in the filter given in (10)-(13). Bayesian estimation of TVTP models via the Gibbs Sampler is also straightforward, and is discussed in Filardo and Gordon (1998). Despite its intuitive appeal, the literature contains relatively few applications of the TVTP model. A notable example of the TVTP framework is found in Durland and McCurdy (1994), Filardo and Gordon (1998) and Kim and Nelson (1998), who study business cycle "duration dependence," or whether the probability of a business cycle phase shift depends on how long the economy has been in the current phase. Other applications include Ang and Bekaert (2002), who model regime-switches in interest rates, and Lo and

Piger (2005), who investigate sources of time-variation in the response of output to monetary policy actions.

The TVTP model is capable of relaxing the restriction that the state variable, S_t, is independent of lagged values of the series, y_t, and thus of lagged values of the disturbance term, ε_t. Kim, Piger and Startz (2003) consider a Markov-switching model in which S_t is also correlated with the contemporaneous value of ε_t, and is thus "endogenous". They model this endogenous switching by assuming that the shock to the probit process in (6), η_t, and ε_t are jointly normally distributed as follows:

$$\begin{bmatrix} \varepsilon_t \\ \eta_t \end{bmatrix} \sim N(0, \Sigma), \quad \Sigma = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}. \tag{17}$$

Kim, Piger and Startz (2003) show that when $\rho \neq 0$, the conditional density in (12) is no longer Gaussian, but can be evaluated analytically. Thus, the likelihood function for the endogenous switching model can be evaluated with simple modifications to the recursive filter in (10)-(13). Tests of the null hypothesis that S_t is exogenous can also be implemented in a straightforward manner. Chib and Dueker (2004) consider endogenous switching as in (17) from a Bayesian perspective.

The extensions listed above are primarily modifications to the stochastic process assumed to drive S_t. A more fundamental extension of (2) is to consider Markov-switching in time series models more complicated than simple autoregressions. An important example of this is a state-space model with Markov-switching parameters. Allowing for Markov-switching in the state-space representation for a time series is particularly interesting because a large number of popular time-series models can be given a state-space representation. Thus, incorporating Markov-switching into a general state-space representation immediately extends the Markov-switching framework to these models.

To aid discussion, consider the following Markov-switching state-space representation for a vector of *R* random variables, $Y_t = (y_{1t}, y_{2t}, ..., y_{Rt})'$, given as follows:

$$\begin{aligned} Y_t &= H_{S_t}{}' X_t + W_t \\ X_t &= A_{S_t} + F_{S_t} X_{t-1} + V_t \end{aligned}, \tag{18}$$

where $X_t = (x_{1t}, x_{2t}, ..., x_{Dt})'$, $W_t \sim N(0, B_{S_t})$ and $V_t \sim N(0, Q_{S_t})$. The parameters of the model undergo Markov switching, and are contained in the matrices $H_{S_t}, B_{S_t}, A_{S_t}, F_{S_t}, Q_{S_t}$. A case of primary interest is when some or all of the elements of X_t are unobserved. This is the case for a wide range of important models in practice, including models with moving average (MA) dynamics, unobserved components (UC) models, and dynamic factor models. However, in the presence of Markov-switching parameters, the fact that X_t is unobserved introduces substantial complications for construction of the likelihood function. In particular, as is discussed in detail in Kim (1994) and Kim and Nelson (1999a), exact construction of the conditional density $f(y_t \mid \Omega_{t-1}; \theta)$ requires that one consider all possible permutations of the entire history of the state variable, S_t, S_{t-1}, $S_{t-2}, \ldots, S_1$. For even moderately sized values of *t*, this quickly becomes computationally infeasible.

To make inference via maximum likelihood estimation feasible, Kim (1994) develops a recursive filter that constructs an approximation to the likelihood function. This filter "collapses" the number of lagged regimes that are necessary to keep track of by approximating a nonlinear expectation with a linear projection. Kim and Nelson (1999a) provide a detailed description of the Kim (1994) filter, as well as a number of examples of its practical use.

If one is willing to take a Bayesian approach to the problem, Kim and Nelson (1998) show that inference can be conducted via the Gibbs Sampler without resorting to approximations. As before, the conditioning features of the Gibbs sampler greatly simplifies the analysis. For example, by conditioning on $\widetilde{S} = \left(S_1, S_2, ..., S_T\right)'$, the model in (18) is simply a linear, Gaussian, state-space model with dummy variables, for which techniques to sample the posterior distribution of model parameters and the unobserved elements of X_t are well established (Carter and Kohn, 1994). Kim and Nelson (1999a) provide detailed descriptions of how the Gibbs Sampler can be implemented for a state-space model with Markov switching.

There are now many applications of state space models with Markov switching in the applied literature. For example, a large literature uses UC models to decompose measures of economic output into trend and cyclical components, with the cyclical component often interpreted as a measure of the business cycle. Until recently, this literature focused on linear representations for the trend and cyclical components (Harvey, 1985; Watson, 1986; Clark, 1987; Morley, Nelson and Zivot, 2003). However, one might think that the processes used to describe the trend and cyclical components might display regime switching in a number of directions, such as that related to the phase of the business cycle or to longer-run structural breaks in productivity growth or volatility. A UC model with Markov switching in the trend and cyclical components can be cast as a Markov-switching state-space model as in (18). Applications of such regime-switching UC models can be found in Kim and Nelson (1999b), Kim and Murray (2002), Kim and Piger (2002), Mills and Wang (2002), and Sinclair (2007). Another primary example of a Markov-switching state-space model is a dynamic factor model with Markov-switching parameters, examples of which are given in Chauvet (1998) and Kim and Nelson (1998). Section VII presents a detailed empirical example of such a model.

VI. Specification Testing for Markov-Switching Models

Our discussion so far has assumed that key choices in the specification of regime-switching models are known to the researcher. Chief among these is the choice of the number of regimes, *N*. However, in practice there is likely uncertainty about the appropriate number of regimes. This section discusses data-based techniques that can be used to select the value of *N*.

To fix ideas, consider a simple version of the Markov-switching model in (2):

$$y_t = \alpha_{S_t} + \varepsilon_t, \qquad (19)$$

where $\varepsilon_t \sim N(0, \sigma^2)$. Consider the problem of trying to decide between a model with $N = 2$ regimes vs. the simpler model with $N = 1$ regimes. The model with one regime is a constant parameter model, and thus this problem can be interpreted as a decision between a model with regime-switching parameters vs. one without. An obvious choice for making this decision is to construct a test of the null hypothesis of $N = 1$ vs. the alternative of $N = 2$. For example, one might construct the likelihood ratio statistic:

$$LR = 2\left(L\left(\hat{\theta}_{MLE(2)}\right) - L\left(\hat{\theta}_{MLE(1)}\right)\right), \tag{20}$$

where $\hat{\theta}_{MLE(1)}$ and $\hat{\theta}_{MLE(2)}$ are the maximum likelihood estimates under the assumptions of $N = 1$ and $N = 2$ respectively. Under the null hypothesis there are three fewer parameters to estimate, α_2, γ_1 and γ_2, than under the alternative hypothesis. Then, to test the null hypothesis, one might be tempted to proceed by constructing a p-value for LR using the standard $\chi^2(3)$ distribution.

However, this final step is not justified, and can lead to very misleading results in practice. In particular, the standard conditions for LR to have an asymptotic χ^2 distribution include that all parameters are identified under the null hypothesis (Davies, 1977). In the case of the model in (19), the parameters γ_1 and γ_2, which determine the transition probabilities p_{ij}, are not identified assuming the null hypothesis is true. In particular, if $\alpha_1 = \alpha_2$, then p_{ij} can take on any values without altering the likelihood function for the observed data. A similar problem exists when testing the general case of N vs. N+1 regimes.

Fortunately, a number of contributions in recent years have produced asymptotically justified tests of the null hypothesis of N regimes vs. the alternative of N+1 regimes. In particular, Hansen (1992) and Garcia (1998) provide techniques to compute asymptotically valid critical values for LR. Recently Carrasco, Hu and Ploberger (2004) have developed an asymptotically optimal test for the null hypothesis of parameter constancy against the general alternative of Markov-switching parameters. Their test is particularly appealing because it does not require estimation of the model under the alternative hypothesis, as is the case with *LR*.

If one is willing to take a Bayesian approach, comparison of models with N vs. N+1 regimes creates no special considerations. In particular, one can proceed by computing standard Bayesian model comparison metrics, such as Bayes Factors or posterior odds ratios. Examples of such comparisons can be found in Chib (1995), Koop and Potter (1999), and Kim and Nelson (2001).

VII. Empirical Example: Identifying Business Cycle Turning Points

This section presents an empirical example demonstrating how the Markov-switching framework can be used to model shifts between expansion and recession phases in the U.S. business cycle. This example is of particular interest for two reasons. First, although Markov-switching models have been used to study a wide variety of topics, their most common application has been as formal statistical models of business cycle phase shifts. Second, the particular model we focus on here, a dynamic factor model with Markov-switching parameters, is of interest in its own right, with a number of potential applications.

The first presentation of a Markov-switching model of the business cycle is found in Hamilton (1989). In particular, Hamilton (1989) showed that U.S. real GDP growth could be characterized as an autoregressive model with a mean that switched between low and high growth regimes, where the estimated timing of the low growth regime corresponded closely to the dates of U.S. recessions as established by the Business Cycle

Dating Committee of the National Bureau of Economic Research (NBER). This suggested that Markov-switching models could be used as tools to identify the timing of shifts between business cycle phases, and a great amount of subsequent analysis has been devoted toward refining and using the Markov-switching model for this task.

The model used in Hamilton (1989) was univariate, considering only real GDP. However, as is discussed in Diebold and Rudebusch (1996), a long emphasized feature of the business cycle is comovement, or the tendency for business cycle fluctuations to be observed simultaneously in a large number of economic sectors and indicators. This suggests that, by using information from many economic indicators, the identification of business cycle phase shifts might be sharpened. One appealing way of capturing comovement in a number of economic indicators is through the use of dynamic factor models, as popularized by Stock and Watson (1989, 1991). However, these models assumed constant parameters, and thus do not model business cycle phase shifts explicitly.

To simultaneously capture comovement and business cycle phase shifts, Chauvet (1998) introduces Markov-switching parameters into the dynamic factor model of Stock and Watson (1989, 1991). Specifically, defining $y_{rt}^* = y_{rt} - \bar{y}_r$ as the demeaned growth rate of the *r*'th economic indicator, the dynamic factor Markov-switching (DFMS) model has the form:

$$y_{rt}^* = \beta_r c_t + e_{rt}. \tag{21}$$

In (21), the demeaned first difference of each series is made up of a component common to each series, given by the dynamic factor c_t, and a component idiosyncratic to each series, given by e_{rt}. The common component is assumed to follow a stationary autoregressive process:

$$\phi(L)(c_t - \mu_{S_t}) = \varepsilon_t . \tag{22}$$

where $\varepsilon_t \sim i.i.d.N(0,1)$. The unit variance for ε_t is imposed to identify the parameters of the model, as the factor loading coefficients, β_r, and the variance of ε_t are not separately identified. The lag polynomial $\phi(L)$ is assumed to have all roots outside of the unit circle. Regime switching is introduced by allowing the common component to have a Markov-switching mean, given by μ_{S_t}, where $S_t = \{1,2\}$. The regime is normalized by setting $\mu_2 < \mu_1$. Finally, each idiosyncratic component is assumed to follow a stationary autoregressive process:

$$\theta_r(L)e_{rt} = \omega_{rt} . \tag{23}$$

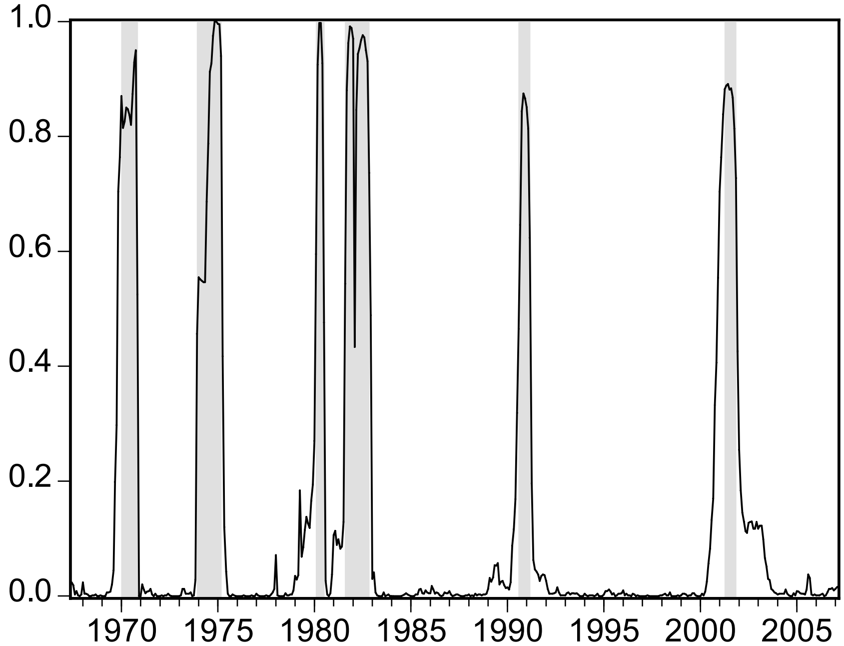
where $\theta_r(L)$ is a lag polynomial with all roots outside the unit circle and $\omega_{rt} \sim N\left(0, \sigma^2_{\omega,r}\right)$.

Chauvet (1998) estimates the DFMS model for U.S. monthly data on non-farm payroll employment, industrial production, real manufacturing and trade sales, and real personal income excluding transfer payments, which are the four monthly variables highlighted by the NBER in their analysis of business cycles. The DFMS model can be cast as a state-space model with Markov switching of the type discussed in Section V. Chauvet estimates the parameters of the model via maximum likelihood, using the approximation to the likelihood function given in Kim (1994). Kim and Nelson (1998) instead use Bayesian estimation via the Gibbs Sampler to estimate the DFMS model.

Here I update the estimation of the DFMS model presented in Kim and Nelson (1998) to a sample period extending from February 1967 through February 2007. For estimation, I use the Bayesian Gibbs Sampling approach, with prior distributions and specification details identical to those given in Kim and Nelson (1998). The figure below displays $P(S_t = 2 | \Psi_T)$ obtained from the Gibbs Sampler, which is the estimated probability that the low growth regime is active. For comparison, the figure also indicates NBER recession dates with shading.

There are two items of particular interest in the figure. First of all, the estimated probability of the low growth regime is very clearly defined, with $P(S_t = 2 | \Psi_T)$ generally close to either zero or one. Indeed, of the 481 months in the sample, only 32 had $P(S_t = 2 | \Psi_T)$ fall between 0.2 and 0.8. Second, $P(S_t = 2 | \Psi_T)$ is very closely aligned with NBER expansion and recession dates. In particular, $P(S_t = 2 | \Psi_T)$ tends to be very low during NBER expansion phases and very high during NBER recession phases.

The figure demonstrates the value added of employing the DFMS model, which considers the comovement between multiple economic indicators, over models considering only a single measure of economic activity. In particular, results for the Markov-switching autoregressive model of real GDP presented in Hamilton (1989) were based on a data sample ending in 1984, and it is well documented that Hamilton’s original model does not perform well for capturing the two NBER recessions since 1984. Subsequent research has found that allowing for structural change in the residual variance parameter (Kim and Nelson, 1999c; McConnell and Perez-Quiros, 2000) or omitting all linear dynamics in the model (Albert and Chib, 1993; Chauvet and Piger, 2003) improves the Hamilton model's performance. By contrast, the results presented here suggest that the DFMS model accurately identifies the NBER recession dates without a need for structural breaks or the restriction of linear dynamics.



Probability of U.S. Recession from Dynamic Factor Markov-Switching Model

In some cases, we might be interested in converting $P(S_t = 2 \mid \Psi_T)$ into a specific set of dates establishing the timing of shifts between business cycle phases. To do so requires a rule for establishing whether a particular month was an expansion month or a recession month. Here we consider a simple rule, which categorizes any particular month as an expansion month if $P(S_t = 2 \mid \Psi_T) \leq 0.5$ and a recession month if $P(S_t = 2 \mid \Psi_T) > 0.5$. The table below displays the dates of turning points between expansion and recession phases (business cycle peaks), and the dates of turning points between recession and expansion phases (business cycle troughs) that are established by this rule. For comparison, the table also lists the NBER peak and trough dates.

The table demonstrates that the simple rule applied to $P(S_t = 2 \mid \Psi_T)$ does a very good job of matching the NBER peak and trough dates. Of the twelve turning points in the sample, the DFMS model establishes eleven within two months of the NBER date. The exception is the peak of the 2001 recession, for which the peak date from the DFMS model is four months prior to that established by the NBER. In comparing peak and trough dates, the DFMS model appears to do especially well at matching NBER trough dates, for which the date established by the DFMS model matches the NBER date exactly in five of six cases.

Dates of Business Cycle Turning Points Produced by NBER and Dynamic Factor Markov-Switching Model

Peaks			Troughs		
DFMS	**NBER**	**Discrepancy**	**DFMS**	**NBER**	**Discrepancy**
Oct 1969	Dec 1969	2M	Nov 1970	Nov 1970	0M
Dec 1973	Nov 1973	-1M	Mar 1975	Mar 1975	0M
Jan 1980	Jan 1980	0M	Jun 1980	Jul 1980	1M
Jul 1981	Jul 1981	0M	Nov 1982	Nov 1982	0M
Aug 1990	Jul 1990	-1M	Mar 1991	Mar 1991	0M
Nov 2000	Mar 2001	4M	Nov 2001	Nov 2001	0M

Why has the ability of Markov-switching models to identify business cycle turning points generated so much attention? There are at least four reasons. First, it is sometimes argued that recession and expansion phases may not be of any intrinsic interest, as they need not reflect any real differences in the economy's structure. In particular, as noted by Watson (2005), simulated data from simple, constant parameter, time-series models, for which the notion of separate regimes is meaningless, will contain episodes that look to the eye like "recession" and "expansion" phases. By capturing the notion of a business cycle phase formally inside of a statistical model, the Markov-switching model is then able to provide statistical evidence as to the extent to which business cycle phases are a meaningful concept. Second, although the dates of business cycle phases and their associated turning points are of interest to many economic researchers, they are not compiled in a systematic fashion for many economies. Markov-switching models could then be applied to obtain business cycle turning point dates for these economies. An example of this is given in Owyang, Piger and Wall (2005), who use Markov-switching models to establish business cycle phase dates for U.S. states. Third, if economic time-series do display different behavior over business cycle phases, then Markov-switching models designed to capture such differences might be exploited to obtain more accurate forecasts of economic activity. Finally, the current probability of a new economic turning point is likely of substantial interest to economic policymakers. To this end, Markov-switching models can be used for "real-time" monitoring of new business cycle phase shifts. Indeed, Chauvet and Piger (2004) provide evidence that Markov-switching models are often quicker to establish U.S. business cycle turning points, particularly at business cycle troughs, than is the NBER. For additional analysis of the ability of regime-switching models to establish turning points in real time, see Chauvet and Piger (2003) and Chauvet and Hamilton (2006).

VIII. Future Directions

Research investigating applied and theoretical aspects of regime-switching models should be an important component of the future research agenda in macroeconomics and econometrics. In this section I highlight three directions for future research that are of particular interest.

To begin, additional research oriented toward improving the forecasting ability of regime-switching models is needed. In particular, given that regime-switching models of economic data contain important deviations from traditional, constant parameter, alternatives, we might expect that they could also provide improved out-of-sample forecasts. However, as surveyed in Clements, Franses and Swanson (2004), the forecasting improvements generated by regime-switching models over simpler alternatives is spotty at best. That this is true is perhaps not completely surprising. For example, the ability of a Markov-switching model to identify regime shifts in past data does not guarantee that the model will do well at detecting regime shifts quickly enough in real time to generate improved forecasts. This is particularly problematic when regimes are short lived. Successful efforts to improve the forecasting ability of Markov-switching models are likely to come in the form of multivariate models, which can utilize additional information for quickly identifying regime shifts.

A second potentially important direction for future research is the extension of the Markov-switching dynamic factor model discussed in Sections V and VII to settings with a large cross-section of data series. Indeed, applications of the DFMS model have been largely restricted to a relatively small number of variables, such as in the model of the U.S. business cycle considered in Section VII. However, in recent years there have been substantial developments in the analysis of dynamic factor models comprising a large number of variables, as in Forni, Hallin, Lippi and Reichlin (2000, 2002, 2005) and Stock and Watson (2002a, 2002b). Research extending the regime-switching framework to such "big data" factor models will be of substantial interest.

Finally, much remains to be done incorporating regime-switching behavior into structural macroeconomic models. A number of recent studies have begun this synthesis by considering the implications of regime-switches in the behavior of a fiscal or monetary policymaker for the dynamics and equilibrium behavior of model economies (Davig, Leeper and Chung, 2004; Davig and Leeper, 2005, 2006; Farmer, Waggoner and Zha, 2006, 2007). This literature has already yielded a number of new and interesting results, and is likely to continue to do so as it expands. Less attention has been paid to reconciling structural models with a list of new "stylized facts" generated by the application of regime-switching models in reduced-form settings. As one example, there is now a substantial list of studies, including Beaudry and Koop (1993), Sichel (1994), Kim and Nelson (1999), Kim and Murray (2002), Hamilton (2005b) and Kim, Morley and Piger (2005), finding evidence that the persistence of shocks to key macroeconomic variables varies dramatically over business cycle phases. However, such an asymmetry is absent from most modern structural macroeconomic models, which generally possess a symmetric propagation structure for shocks. Research designed to incorporate and explain business cycle asymmetries and other types of regime-switching behavior inside of structural macroeconomic models will be particularly welcome.

IX. Bibliography

Books and Reviews

Clements, M.P., Franses, P.H. and N.R. Swanson, 2004, Forecasting Economic and Financial Time-Series with Non-Linear Models, *International Journal of Forecasting* 20, 169-183.

Granger, C.W.J. and T. Teräsvirta, 1993, Modelling Nonlinear Economic Relationships. Oxford: Oxford University Press.

Hamilton, J.D., 1994, Time Series Analysis, Princeton, NJ: Princeton University Press.

Hamilton, J.D. and B. Raj, 2002, New Directions in Business Cycle Research and Financial Analysis, *Empirical Economics,* 27, 149-162.

Hamilton, J.D., 2005a, Regime-Switching Models, in *New Palgrave Dictionary of Economics*, *2nd edition*, eds. S. Durlauf and L. Blume, Palgrave McMillan Ltd.

Kim, C.-J. and C. Nelson, 1999a, *State-Space Models with Regime Switching: Classical and Gibbs-Sampling Approaches with Applications*. Cambridge: MIT Press.

Krolzig, H.-M., 1997, Markov-Switching Vector Autoregressions : Modelling, Statistical Inference, and Application to Business Cycle Analysis, Berlin: Springer.

Tong, H., 1983, Threshold models in non-linear time series analysis, Lecture Notes in Statistics, No. 21, Heidelberg: Springer.

Zellner, A., 1971, An Introduction to Bayesian Inference in Econometrics, New York: John Wiley and Sons, Inc.

Primary Literature

Albert, J. and S. Chib, 1993, Bayes Inference via Gibbs Sampling of Autoregressive Time Series Subject to Markov Mean and Variance Shifts, *Journal of Business and Economic Statistics* 11, 1-15.

Ang, A. and G. Bekaert, 2002, Regime Switches in Interest Rates, *Journal of Business and Economic Statistics*, 20, 163-182.

Beaudry, P. and G. Koop, 1993, Do Recessions Permanently Change Output?, *Journal of Monetary Economics* 31, 149-163.

Campbell, S.D., 2002, Specification Testing and Semiparametric Estimation of Regime Switching Models: An Examination of the US Short Term Interest Rate, Brown University Department of Economics Working Paper #2002-26.

Carrasco, M., Hu, L. and W. Ploberger, 2004, Optimal Test for Markov Switching, Working paper, University of Rochester.

Carter, C.K. and P. Kohn, 1994, On Gibbs Sampling for State Space Models, *Biometrica,* 81, 541-553.

Chauvet, M., 1998, "An Econometric Characterization of Business Cycle Dynamics with Factor Structure and Regime Switching," *International Economic Review,* 39, 969-996.

Chauvet, M. and J. Hamilton, 2006, "Dating Business Cycle Turning Points," in *Nonlinear Time Series Analysis of Business Cycles,* eds. C. Milas, P. Rothman, and D. van Dijk, Elsevier, North Holland.

Chauvet, M. and J. Piger, 2003, "Identifying Business Cycle Turning Points in Real Time," *Federal Reserve Bank of St. Louis Review* 85, 47-61.

Chauvet, M. and J. Piger, 2004, "A Comparison of the Real-Time Performance of Business Cycle Dating Methods," *Journal of Business and Economic Statistics,* (in press).

Chib, S., 1995, Marginal Likelihood from the Gibbs Output. *Journal of the American Statistical Association* 90, 1313-1321.

Chib, S., 1998, Estimation and Comparison of Multiple Change-Point Models. *Journal of Econometrics* 86, 221-241.

Chib, S. and M. Dueker, 2004, Non-Markovian Regime Switching with Endogenous States and Time Varying State Strengths, Federal Reserve Bank of St. Louis Working Paper #2004-030A.

Clark, P.K., 1987, The Cyclical Component of U.S. Economic Activity, *Quarterly Journal of Economics* 102, 797-814.

Cosslett, S. R. and L.-F. Lee, 1985, "Serial Correlation in Discrete Variable Models," Journal of Econometrics 27, 79-97.

Davies, R.B., 1977, "Hypothesis testing when a nuisance parameter is present only under the alternative," Biometrika 64, 247-254.

Davig, T. and E. Leeper, 2005, "Generalizing the Taylor Principle," *American Economic Review*, forthcoming.

Davig. T, and E. Leeper, 2006, “Endogenous Monetary Policy Regime Change,” in *International Seminar on Macroeconomics*, eds. L. Reichlin and K.D. West, MIT Press, Cambridge.

Davig, T., E. Leeper and H. Chung, 2004, “Monetary and Fiscal Policy Switching,” *Journal of Money, Credit and Banking,* forthcoming.

Diebold, F.X., Lee, J-H. and G. Weinbach, 1994, Regime Switching with Time-Varying Transition Probabilities, in *Non-stationary Time Series Analysis and Cointegration,* ed. C. Hargreaves, Oxford University Press, Oxford, U.K.

Diebold, F.X. and G.D. Rudebusch, 1996, ‘Measuring business cycles: A modern perspective’, *The Review of Economics and Statistics*, 78, 67-77.

Dueker, M., 1997, “Markov Switching in GARCH Processes and Mean-Reverting Stock-Market Volatility,” Journal of Business and Economic Statistics 15, 26-34.

Durland, J.M. and T.H. McCurdy, 1994, Duration Dependent Transitions in a Markov Model of U.S. GNP Growth, *Journal of Business and Economic Statistics* 12, 279-288.

Evans, M. and P. Wachtel, 1993, “Inflation Regimes and the Sources of Inflation Uncertainty,” *Journal of Money, Credit, and Banking* 25, 475-511.

Farmer, R.E.A., D.F. Waggoner and T. Zha, 2006, “Indeterminacy in a Forward Looking Regime Switching Model,” NBER working paper no. 12540.

Farmer, R.E.A., D.F. Waggoner and T. Zha, 2007, “Understanding the New-Keynesian Model when Monetary Policy Switches Regimes,” NBER working paper no. 12965.

Filardo, A.J., 1994, Business-Cycle Phases and Their Transitional Dynamics, *Journal of Business and Economic Statistics,* 12, 299-308.

Filardo, A.J. and S.F. Gordon, 1998, “Business Cycle Durations,” Journal of Econometrics 85, 99-123.

Forni, M., Hallin, M., Lippi, F. and L. Reichlin, 2000, The Generalized Dynamic Factor Model: Identification and Estimation, *Review of Economics and Statistics* 82, 540-554.

Forni, M., Hallin, M., Lippi, F. and L. Reichlin, 2002, The generalized dynamic factor model: consistency and convergence rates, *Journal of Econometrics* 82 540-554.

Forni, M., Hallin, M., Lippi, F. and L. Reichlin, 2005, The generalized dynamic factor model: one-sided estimation and forecasting, *Journal of the American Statistical Association* 100, 830-840.

Garcia, R., 1998, Asymptotic null distribution of the likelihood ratio test in Markov switching models, International Economic Review 39, 763-788.

Garcia, R. and P. Perron, 1996, An Analysis of the Real Interest Rate under Regime Shifts, *Review of Economics and Statistics,* 78, 111-125.

Garcia, R. and H. Schaller, 2002, Are the Effects of Monetary Policy Asymmetric?, *Economic Inquiry* 40, 102-119.

Gelfand, A. E. and A.F.M. Smith, 1990, "Sampling-Based Approaches to Calculating Marginal Densities," *Journal of the American Statistical Association,* 85, 398-409.

Geman, S. and D. Geman, 1984, "Stochastic Relaxation, Gibbs Distributions and the Bayesian Restoration of Images," *IEEE Transactions on Pattern Analysis and Machine Intelligence* 6, 721-741.

Goldfeld, S.M. and R.E. Quandt, 1973, A Markov Model for Switching Regressions, *Journal of Econometrics,* 1, 3-16.

Guidolin, M. and A. Timmermann, 2005, Economic Implications of Bull and Bear Regimes in UK Stock and Bond Returns, *Economic Journal* 115, 111-143.

Hamilton, J.D., 1989, A New Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle, *Econometrica* 57, 357-384.

Hamilton, J.D., 1991, A Quasi-Bayesian Approach to Estimating Parameters for Mixtures of Normal Distributions, *Journal of Business and Economic Statistics,* 9, 27-39.

Hamilton, J.D., 2005b, "What's Real About the Business Cycle?", Federal Reserve Bank of St. Louis Review, 87, 435-452.

Hamilton, J.D. and G. Lin, 1996, "Stock Market Volatility and the Business Cycle, *Journal of Applied Econometrics* 11, 573-593.

Hamilton, J.D. and R. Susmel, 1994, Autoregressive Conditional Heteroskedasticity and Changes in Regime, *Journal of Econometrics* 64, 307-333.

Hamilton, J.D. Waggoner, D.F. and T. Zha, 2004, Normalization in Econometrics, *Econometric Reviews* (in press).

Hansen, B.E., 1992, The likelihood ratio test under nonstandard conditions: Testing the Markov switching model of GNP, Journal of Applied Econometrics 7, S61-S82.

Harvey, A.C., 1985, Trends and Cycles in Macroeconomic Time Series, *Journal of Business and Economic Statistics* 3, 216-227.

Juang, B.-H., and L.R. Rabiner, 1985, “Mixture Autoregressive Hidden Markov Models for Speech Signals,” IEEE Transactions on Acoustics, Speech, and Signal Processing ASSP-30, 1404-1413.

Kaufmann, S., 2002, Is there an Asymmetric Effect of Monetary Policy Over Time? A Bayesian Analysis using Austrian Data, *Empirical Economics* 27, 277-297.

Kim, C.-J., 1994, Dynamic linear models with Markov-switching, *Journal of Econometrics* 60, 1-22.

Kim, C.-J., 2004, Markov-Switching Models with Endogenous Explanatory Variables, *Journal of Econometrics* 122, 127-136.

Kim, C.-J., Morley, J. and C. Nelson, 2002, Is there a Positive Relationship between Stock Market Volatility and the Equity Premium?, *Journal of Money Credit and Banking* 36, 339-360.

Kim, C.-J., Morley, J. and J. Piger, 2005, Nonlinearity and the Permanent Effects of Recessions, *Journal of Applied Econometrics* 20, 291-309.

Kim, C.-J. and C.J. Murray, 2002, Permanent and transitory components of recessions, Empirical Economics 27, 163-183.

Kim, C.-J. and C.R. Nelson, 1998, “Business Cycle Turning Points, a New Coincident Index, and Tests of Duration Dependence Based on a Dynamic Factor Model with Regime Switching,” *The Review of Economics and Statistics,* 80, 188-201.

Kim, C.-J. and C.R. Nelson, 1999b, Friedman’s Plucking Model of Business Fluctuations: Tests and Estimates of Permanent and Transitory Components, *Journal of Money, Credit and Banking* 31, 317-34.

Kim, C.-J. and C.R. Nelson, 1999c, “Has the U.S. economy become more stable? A Bayesian approach based on a Markov-switching model of the business cycle,” *Review of Economics and Statistics* 81, 608-616.

Kim, C.-J. and C.R. Nelson, 2001, A Bayesian approach to testing for Markov-switching in univariate and dynamic factor models, *International Economic Review*, 42, 989-1013.

Kim, C.-J. and J. Piger, 2002, ‘Common stochastic trends, common cycles, and asymmetry in economic fluctuations’, *Journal of Monetary Economics,* 49, 1189-1211.

Kim, C.-J., Piger, J. and R. Startz, 2003, “Estimation of Markov Regime-Switching Regression Models with Endogenous Switching,” Federal Reserve Bank of St. Louis working paper # 2003-015C.

Kim, C.-J., Piger, J. and R. Startz, 2007, "The Dynamic Relationship Between Permanent and Transitory Components of U.S. Business Cycles," *Journal of Money, Credit and Banking* 39, 187-204.

Koop, G. and S.M. Potter, 1999, "Bayes Factors and Nonlinearity: Evidence from Economic Time Series," Journal of Econometrics 88, 251-281.

Lo, M. and J. Piger, 2005, "Is the Response of Output to Monetary Policy Asymmetric? Evidence from a Regime-Switching Coefficients Model," *Journal of Money, Credit and Banking* 37, 865-887.

McConnell, M.M. and G.P. Quiros, 2000, "Output Fluctuations in the United States: What has Changed Since the Early 1980s?" *American Economic Review* 90, 1464-1476.

Mills, T.C. and P. Wang, 2002, Plucking Models of Business Cycle Fluctuations: Evidence from the G-7 Countries, *Empirical Economics* 27, 255-276.

Morley, J.C., Nelson, C.R. and E. Zivot, 2003, Why Are the Beveridge-Nelson and Unobserved-Components Decompositions of GDP So Different? *The Review of Economics and Statistics* 85, 235-243.

Öcal, N. and D.R. Osborn, 2000, Business cycle non-linearities in UK consumption and production, Journal of Applied Econometrics 15, 27-44.

Owyang, M.T., Piger, J. and H.J. Wall, 2005, Business Cycle Phases in U.S. States, *Review of Economics and Statistics* 87, 604-616.

Pesaran, M.H. and S. M. Potter, 1997, A floor and ceiling model of U.S. output, Journal of Economic Dynamics and Control, 21, 661-695.

Poritz, A.B., 1982, "Linear Predictive Hidden Markov Models and the Speech Signal," Acoustics, Speech and Signal Processing, IEEE Conference on ICASSP '82, vol. 7, 1291-1294.

Potter, S.M., 1995, A Nonlinear Approach to U.S. GNP, *Journal of Applied Econometrics* 10, 109-125.

Potter, S.M., 1999, Nonlinear Time Series Modelling: An Introduction, *Journal of Economic Surveys,* 13, 505-528.

Rabiner, L.R., 1989, "A Tutorial on Hidden Markov Models and Selected Applications in Speech Recognition," Proceedings of the IEEE 77, 257-286.

Rapach, D.E. and M.E. Wohar, 2002, Regime Changes in International Real Interest Rates: Are They a Monetary Phenomenon? *Journal of Money, Credit and Banking* 37, 887-906.

Ravn, M. and M. Sola, 2004, Asymmetric Effects of Monetary Policy in the United States, *Federal Reserve Bank of St. Louis Review*, 86, 41-60.

Sichel, D. E., 1994, "Inventories and the three phases of the business cycle," *Journal of Business and Economic Statistics* 12, 269-277.

Sims, C. and T. Zha, 2006, Were there Regime Changes in U.S. Monetary Policy?, *American Economic Review* 96, 54-81.

Sinclair, T., 2007, Asymmetry in the Business Cycle: A New Unobserved Components Model, George Washington University working paper.

Stock, J.H. and M.W. Watson, 1989, "New Indexes of Coincident and Leading Economic Indicators," *NBER Macroeconomics Annual*, 4, 351-393.

Stock, J.H. and M.W. Watson, 1991, "A Probability Model of the Coincident Economic Indicators," in *Leading Economic Indicators: New Approaches and Forecasting Records,* eds. K. Lahiri and G.H. Moore, Cambridge: Cambridge University Press.

Stock, J.H. and M.W. Watson, 2002a, "Forecasting using principal components from a large number of predictors", *Journal of the American Statistical Association* 97:1167–1179.

Stock, J.H. and M.W. Watson, 2002b, Macroeconomic Forecasting Using Diffusion Indexes (with James H. Stock), *Journal of Business and Economic Statistics*, April 2002, Vol. 20 No. 2, 147-162,

Tanner, M. and W. Wong, 1987, "The Calculation of Posterior Distributions by Data Augmentation," *Journal of the American Statistical Association* 82, 528-550.

Tiao, G.C. and R.S. Tsay, 1994, Some advances in non-linear and adaptive modeling in time-series analysis, *Journal of Forecasting* 13, 109-131.

Turner, C. M., Startz, R. and C.R. Nelson, 1989, A Markov Model of Heteroskedasticity, Risk, and Learning in the Stock Market, *Journal of Financial Economics*, 25, 3-22.

van Dijk, D. and P.H. Franses, 1999, Modeling multiple regimes in the business cycle, Macroeconomic Dynamics 3, 311-340.

Watson, M.W., 1986, Univariate Detrending Methods with Stochastic Trends, *Journal of Monetary Economics* 18, 49-75.

Watson, M.W., 2005, Comment on James D. Hamilton's 'What's Real about the Business Cycle?', *Federal Reserve Bank of St. Louis Review* 87, 453-458.