Robust Identification of Business Cycle Turning Points

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Abstract: The global pandemic created extreme swings in macroeconomic data that complicate statistical analysis. This included an uncharacteristically short recession that was unprecedented in the monthly growth rates observed during the recession. This paper first shows that the performance of standard Markov-switching models for classifying business cycle phases and their turning points drastically deteriorates when the pandemic recession is included in the data sample. Further, simulations reveal that this deterioration would have likely happened even if the pandemic recession had been far less severe. An alternative, robust, approach to Markov-switching models for classifying business cycle phases is proposed based on Markov-switching quantile models. Specifically, I investigate the performance of dynamic factor Markov switching (DFMS) models that are specified in terms of switching quantiles rather than switching means. I show that these quantile DFMS models are capable of classifying recessions, both retrospectively and in real time, even when the estimation sample contains outlier recessions.

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1 Introduction

In his seminal publication, Hamilton (1989) proposed an autoregressive model in which the parameters of the process experience discrete shifts between regimes. The regime shifts are unobserved, but are assumed to be governed by a Markov-process. The timing of regime change can then be estimated from the data using a nonlinear, recursive filter. In an application to quarterly U.S. real GNP growth, Hamilton (1989) showed that an autoregressive process with Markov-switching between a high mean growth regime and a low mean growth regime produced estimated timing of regimes that align closely with expansion and recession regimes as defined by the National Bureau of Economic Research (NBER). In the words of the classification literature, the Hamilton (1989) univariate model served as an unsupervised classifier of expansion and recession quarters that yielded a close match to the classification produced by the NBER.

Based on this success at business cycle classification, Markov-switching (MS) models quickly became the dominant model-based approach to determine the dates of U.S. business cycle phases. A wide variety of extensions to the original Hamilton (1989) model were considered, including higher frequency data (Boldin (1994)), incorporation of more than two regimes (Boldin (1996); Sichel (1994); Hamilton (2005); Kim et al. (2005)), and allowing the probability of a regime shift to depend on exogenous predictors (Diebold et al. (1994); Filardo (1994)). A particularly important extension was to include multivariate data by modeling Markov-switching in a dynamic factor estimated from a set of coincident economic data. Such a "Dynamic-Factor Markov-Switching" (DFMS) model was proposed by Diebold and Rudebusch (1996) as a two-step procedure in which a dynamic factor is first estimated, and then subsequently fit to a Markov-switching autoregressive process. This was later extended to joint maximum-likelihood estimation by Chauvet (1998) using the nonlinear filter developed in Kim (1994), and to joint Bayesian estimation in Kim and Nelson (1998). Markov-switching models also became widely used to classify business cycle phases in non-U.S. data and on sub-national data (Smith and Summers (2005); Owyang et al. (2005);

Hamilton and Owyang (2012); Owyang et al. (2008)).

In addition to their usefulness for historical business cycle classification, Markov-switching models have been shown to work well for real time nowcasting of new business cycle turning points. In the United States, the NBER tends to announce new turning points with a considerable lag, prioritizing accuracy over timeliness. In other countries or in sub-national data, there may not be any real-time announcement of new phases at all. In real-time classification experiments with U.S. data, Markov-switching models have been shown to identify new phases more quickly than the NBER with few false positives. (Layton (1996); Chauvet and Piger (2003); Chauvet and Piger (2008); Chauvet and Hamilton (2006); Hamilton (2011).) Further, the real-time performance of Markov-switching models has been show to compare favorably to other commonly used classification methods, even those supervised by the NBER classification (Chauvet and Piger (2008); Piger (2020).)

The favorable performance of Markov-switching models for dating U.S. business cycles, and especially the DFMS variant, was largely consistent across the decades since the publication of Hamilton (1989). However, the inclusion of economic data from the COVID-19 pandemic recession and subsequent recovery has substantially deteriorated the ability of Markov-switching models to classify business cycle phases generally. The left panel of Figure 1 shows the estimated probability of the low mean growth regime taken from a DFMS model estimated over the period from February 1967 through February 2020 along with shading indicating the dates of NBER recession months.[1] As is clear from this graph, the Markov-switching model does a very good job of matching the NBER classification, with the probability of the low mean growth regime very close to 0% during NBER expansion months and well above 50% during NBER recession months. The right panel of Figure 1 shows the estimated probability of the low mean growth regime when the estimation period is extended by just two months, through April 2020. Now, the Markov-switching model captures only the pandemic recession, and no longer captures any other recessions over the sample period.

[1]Details of this DFMS model will be presented in Sections 2-3.

The reason for this sharp deterioration in performance is the outsized effect that the pandemic era data has on the estimated mean growth rate in the low growth regime. Figure 2 shows expanding window estimates of the mean growth rate in the low growth and high growth regime. The first estimates are produced for a window that ends in September 1976, and subsequent estimates are produced for windows expanded one month at a time through August 2025.[2] As is clear from the graph, the estimate of the low mean growth and high mean growth parameters are quite stable over the expanding windows through very early 2020. This is a key reason for the observed stability of the model's recession classification performance across the literature of the 3 decades following the publication of Hamilton (1989). However, as the pandemic-era data enters the sample, the estimated low growth mean parameter falls by a factor of 5 with only two additional months, March and April of 2020, added to the sample. With the low mean growth parameter now so extreme, the model classifies only the pandemic era recession months as being in the low growth regime.

Although the growth rates of the data underlying the DFMS model observed in the pandemic recession were severe, the extreme effect of the pandemic recession on the estimate of mean growth in the low growth regime is perhaps surprising, given that the two months of data added represented only 0.3% of the data sample used in estimation.The reason for the extremely large effect of this outlier recession is that the Markov-switching model is simultaneously estimating both the low growth mean parameter and the timing of the low growth regime itself. In other words, the model has flexibility in what it labels a low growth regime. Because of this, the pandemic recession not only affects the estimated mean growth in the low growth regime for a given classification of low growth regimes, but also affects the classification of when low growth regimes occur. This creates both an intensive margin effect on parameter estimates – mean growth estimates are lowered because the pandemic data was an outlier among the existing classification of low growth regimes – as well as an extensive margin effect – which months are classified as low growth regimes changes to focus

[2]Estimates are produced using real-time vintage data sets that are described in more detail in section 3

more on the outlier recession, which pushes parameter estimates further toward the data observed during the outlier recession. Returning to Figure 1 we can see that this extensive margin effect is severe, as only the pandemic era recession is labeled a low growth regime following the inclusion of data from March and April 2020.

Although the impact of the pandemic recession on the parameters of Markov-switching models is outsized, they are by no means the only macro-econometric models affected by pandemic-era data. For example, the parameters and forecasts of workhorse vector-autoregressive (VAR) models were strongly affected by the inclusion of pandemic-era data. There is a small, but still growing literature studying how best to include pandemic-era data when estimating macro-econometric models. [3] However, the incorporation of pandemic-era data into Markov-switching models for the purpose of dating business cycles has, to the best of my knowledge, not been studied.

One apparent solution is to simply drop, or otherwise nullify, the observations during and immediately following the pandemic recession. While this has been argued for in certain contexts, e.g. Schorfheide and Song (2024), for identifying business cycles this would ignore the relevant economic event of interest, which is the recession. Such an approach also leaves the modeler exposed to future outlier regimes. While one could make the case that a recession as severe as the pandemic recession is very unlikely to occur again, it is possible that outlier recessions of far lower magnitude could upset the performance of the model. Indeed, in Section 4 I show that an outlier recession that was much less severe than the pandemic recession would have had led to a similar deterioration in the classification ability of Markov-switching models. Thus, if one wishes to continue with Markov-switching models as classifiers of business cycle regimes, it seems prudent to develop an approach that is robust to outlier regimes.

Rather than dropping observations, another strand of literature investigates macro-econometric models with added volatility mechanisms to capture the extreme variations

[3] See as examples, Lenza and Primiceri (2022), Schorfheide and Song (2024), Ng (2021), Carriero et al. (2024), Granados and Parra-Amado (2024), and Morley et al. (2023).

in data around the pandemic recession. This includes mechanisms that are specific to the COVID-19 pandemic (e.g. Lenza and Primiceri (2022) and Morley et al. (2023)), as well as approaches designed to capture outlier events more generally. As an example of the latter, Carriero et al. (2024) propose modifications to a standard stochastic volatility process to make it more robust to short bursts of high volatility. This approach is attractive as it can provide improved measures of uncertainty and should provide a mechanism to robustify the model against future such variations. However, for the purposes of dating business cycles, this approach also has the potential to wash out the economic event of interest, recessions, into the volatility component.

In this paper I consider an alternative approach for robust classification of business cycle phases based on quantile regression. Following Hamilton (1989), Markov-switching models for business cycle dating have incorporated Markov-switching into the mean (or conditional mean) of a process, and are estimated via likelihood-based techniques with normally distributed errors. These assumptions leave the estimated model vulnerable to outlier regimes, as fitting the outlier regime receives outsized weight in the associated quadratic loss function. In this paper I instead assume Markov-switching in a quantile of the process, for example the median, where the model is estimated assuming the linear loss function used for estimation of quantile regressions. These model assumptions hold the promise of an estimated model that is more robust to outlier regimes.

I find that Markov-switching quantile (MSQ) models provide an accurate classification of NBER recession dates when applied to an estimated dynamic factor of U.S. macroeconomic data, and this accuracy is robust to the inclusion of data from the pandemic. This is in contrast to the Markov-switching mean (MSM) model, which becomes solely focused on the pandemic recession when pandemic-era data is included in estimation. Interestingly, the best historical classification performance is recorded by MSQ models at quantiles below the median, and specifically around the 30th percentile. Further, simulations reveal that MSM models would experience significantly deteriorated performance for regimes that are outliers to a far lesser extent than the pandemic recession regime, which increases the relevance of developing Markov-switching models that are robust to outlier regimes.

I also consider the performance of the MSQ model for dating business cycles in real time by extending the real-time analysis of Chauvet and Piger (2008) to MSQ models. This analysis uses appropriate vintages of data and thus accurately represents the use of these models in real time. The MSQ model provides accurate and timely classification of the recessions in the real-time evaluation period, which spans from 1977 to 2025. Further, in simulations I show that if the MSM model were used to detect recessions in real time going forward, it would only detect about 50% of future recessions, reflecting the outsized effect of the pandemic recession on parameter estimates in the MSM model. By contrast, the MSQ model would continue to provide accurate classification in simulations of future data. Finally, I show that the improved performance of the MSQ model over the MSM model in the presence of pandemic data extends to business cycle classification in the United Kingdom.

To date, there is only a small literature that has investigated Markov-switching quantile regressions. Liu (2016) and Liu and Luger (2018) propose the Markov-switching quantile regression and develop an approach to Bayesian estimation via the Gibbs Sampler. These papers provide applications to U.S. financial markets. Several recent papers have provided extensions, including Rakpho et al. (2018) and Montes-Rojas et al. (2024), with associated applications to financial market series and U.S. inflation.

The remainder of this paper is organized as follows. Section 2 lays out the MSM and MSQ models and discusses estimation, while Section 3 describes the real-time dataset. Section 4 shows the historical classification results for the MSM and MSQ models applied to datasets that are and are not affected by the pandemic-era data. This section also provides simulation evidence on the extent to which outlier regimes can affect the MSM and MSQ model performance more generally. Section 5 presents results from the real-time analysis, while Section 6 presents results for the United Kingdom. Section 7 concludes.

2 Markov-Switching Models for Business Cycle Classification

This section describes the Markov-switching models used for business cycle phase classification in this paper and how they can be estimated via Bayesian methods. Suppose Y_t is a scalar measure of economic activity providing the signal for business cycle phase classification. Common univariate examples are payroll employment growth for monthly data and Gross Domestic Product (GDP) growth for quarterly data. For multivariate data, the DFMS framework sets Y_t to an estimated latent factor extracted from a multivariate dataset of macroeconomic variables. Consider the following time-series process:

$$\begin{aligned} Y_t &= \alpha^m_{S^m_t} + \sigma^m \varepsilon_t, \\ S^m_t &= \{0, 1\}, \end{aligned} \tag{1}$$

where ε_t is a covariance-stationary process that is independent of S^m_t. In this model, the parameter $\alpha^m_{S^m_t}$ switches between α^m_0 and α^m_1, where the value of $\alpha^m_{S^m_t}$ at time t is determined by the regime-indicator variable S^m_t. If we further assume that $E(\varepsilon_t) = 0$, then $\alpha^m_{S^m_t}$ is the regime-dependent mean of Y_t:

$$E(Y_t | S^m_t) = \alpha^m_{S^m_t}$$

To normalize the regimes, I assume $\alpha^m_1 < \alpha^m_0$ without loss of generality. In the usual case where Y_t represents a growth rate, this means $S^m_t = 1$ has the interpretation as the low growth regime.

The regime indicator S^m_t is generally considered to be unobserved by the econometrician, and a time-series process is assumed for S^m_t to facilitate estimation. Following Hamilton (1989), a common assumption is that S^m_t is a first-order two-state Markov process with transition probabilities $\Pr\left(S^m_t = i | S^m_{t-1} = j\right) = P^m_{ij}$. With this assumption, the model becomes a “Markov-switching” model. Finally, to build a likelihood function, I will assume that $\varepsilon_t \sim$ i.i.d. $N(0,1)$, which is the typical assumption in applications to modeling business cycles. In this case Y_t is i.i.d. normally distributed inside of each regime, with probability density function:

$$f\left(Y_t|S_t^m\right) \propto \frac{1}{\sigma^m} e^{-\frac{1}{2(\sigma^m)^2}\left(y_t-\alpha^m_{S_t^m}\right)^2} \tag{2}$$

In addition to the regime-switching mean, this model will indirectly imply regime-switching in the quantiles of Y_t, as the quantiles of the regime-specific normal distributions will differ. However, if our interest is on estimating regime-switching behavior for specific quantiles, we would likely prefer to estimate the regime-switching quantiles directly and individually, as this will place far less restriction on the behavior of the regime-switching across quantiles. Consider the following analog to Equation (1) for modeling the τ-th quantile of Y_t:

$$\begin{aligned} Y_t &= \alpha^\tau_{S_t^\tau} + \sigma^\tau \omega_t^\tau, \\ S_t^\tau &= \{0,1\}. \end{aligned} \tag{3}$$

In this model, S_t^τ again follows a first-order two-state Markov process with associated transition probabilities P_{ij}^τ. The disturbance ω_t^τ is a covariance stationary process with distribution such that $\Pr\left(\omega_t^\tau < 0\right) = \tau$. This implies:

$$\Pr\left(Y_t < \alpha^\tau_{S_t^\tau} | S_t^\tau\right) = \tau$$

In words, the parameters α_0^τ and α_1^τ are the regime dependent τ-th conditional quantiles of Y_t, which are again normalized by assuming $\alpha_1^\tau < \alpha_0^\tau$. To construct a likelihood function, I will follow the common choice in the quantile regression literature and assume an asymmetric Laplace (AL) density for ω_t^τ, with location parameter $m = 0$, scale parameter $\lambda = 1$, and quantile parameter τ:[4]

$$\omega_t^\tau \sim AL\left(0, 1, \tau\right)$$

[4] The use of the asymmetric Laplace density for Bayesian estimation of quantile regression was first introduced by Yu and Moyeed (2001).

Given this assumption, Y_t has the following regime-dependent density:

$$Y_t|S_t^\tau \sim AL\left(\alpha^\tau_{S_t^\tau}, \sigma^\tau, \tau\right),$$

with associated probability density function:

$$f\left(Y_t|S_t^\tau\right) \propto \frac{1}{\sigma^\tau} e^{-\rho\left(y_t - \alpha^\tau_{S_t}\right)},$$

where, $\rho\left(Y_t - \alpha^\tau_{S_t}\right)$ is the so called "check" or "pinball" loss function:

$$\rho\left(Y_t - \alpha^\tau_{S_t}\right) = \begin{cases} \tau\left|Y_t - \alpha^\tau_{S_t}\right| & \text{if } \left(Y_t - \alpha^\tau_{S_t}\right) \geq 0 \\ (1-\tau)\left|Y_t - \alpha^\tau_{S_t^\tau}\right| & \text{if } \left(Y_t - \alpha^\tau_{S_t^\tau}\right) < 0 \end{cases} \tag{4}$$

From comparison of Equations (2) and (4), one can see how estimating a regime-switching quantile is more robust to outlier regimes than estimating a regime-switching mean. When estimating a regime-switching quantile the loss is linear, specifically in $\left|Y_t - \alpha^\tau_{S_t^\tau}\right|$. When estimating a regime-switching mean the loss is instead quadratic, being in $(Y_t - \alpha^m_{S_t})^2$. This leaves the regime-switching mean model susceptible to large outlier regimes. For example, an outlier regime corresponding to a very low sequence of Y_t will generate large negative movement in the estimate of α^m_1, which will in turn lower the estimated probability that $S_t^m = 1$ outside of the outlier regime.

These models could be complicated on various dimensions, such as allowing for additional sources of dynamics, which would improve in-sample fit. For example, the original Hamilton (1989) Markov-switching model for U.S. GNP growth was an autoregressive model with Markov-switching in mean. This model then contained a source of nonlinear dynamics through the Markov process, and linear dynamics through the autoregressive process. However, our goal is not to maximize the fit of Y_t, but instead to classify economic data into high and low growth regimes, with the ultimate goal of having this classification match traditional definitions of business cycle phases. A prolonged period of negative growth that is driven by linear propagation of a negative shock would almost certainly be considered a recession by traditional definitions, but might not be classified as a low growth regime by a Markov-switching model that also contained a source of linear dynamics. Thus, to maximize the chance of capturing business cycle phase with the Markov-switching classification, I follow Owyang et al. (2005) and model all serial correlation in Y_t as deriving from the nonlinear Markov-switching component.[5]

For a given choice of Y_t, the Markov-switching mean model in (1), hereafter MSM, and the Markov-switching quantile models in (3), hereafter MSQ(τ) will serve as our comparison models for classifying business cycles. To estimate the parameters of these models, and to provide inference on the unobserved regime indicator, I take a Bayesian approach. For the MSM model I use a Gibbs Sampler based on Kim and Nelson (1998). For the MSQ(τ) model, I use the Gibbs Sampler of Liu and Luger (2018), which builds on the Gibbs Sampler developed in Kozumi and Kobayashi (2011) and Khare and Hobert (2012) for quantile regression. This Gibbs Sampler is presented in the appendix. For both models I obtain draws from the posterior distribution of α_0^h and α_1^h, as well as from the posterior distribution of S_t^h, $h \in \{m, \tau\}$, which forms the basis for our classification of regimes.

Bayesian estimation requires priors for the model parameters. Given the large number of specific models considered, here I use identical priors for the analogous parameters of the MSM and all MSQ(τ) models. I use a mix of informative and non-informative priors, where the informative priors are reasonably diffuse. The prior densities are described in more detail in the Appendix, and are summarized in Table 1.

I conclude this section by discussing measurement of Y_t, which contains the data signal used for classification of U.S. business cycle phases. There is now extensive evidence that multivariate information is helpful for classifying business cycle regimes that match the NBER business cycle chronology. As discussed above, the most common framework for incorporating multivariate information is the DFMS model. Here I follow Diebold and Rude-

[5]There is also evidence that once Markov regime-switching is accounted for, there is little remaining serial correlation in measures of U.S. economic output. See, for example, Camacho and Perez-Quiros (2007).

busch (1996) and define Y_t as an estimated dynamic factor of monthly U.S. macroeconomic variables. Specifically, let X_t be a $k \times 1$ vector of macroeconomic variables, and $X_{i,t}$ is the i^{th} element of X_t. I assume the following factor structure for $X_{i,t}$:

$$\begin{aligned} X_{i,t} &= \psi_i Y_t^\star + u_{i,t} \\ Y_t^\star &= \phi_1 Y_{t-1}^\star + \phi_2 Y_{t-2}^\star + e_t \end{aligned} \tag{5}$$

where $Y_t^\star$ is a scalar, unobserved, dynamic factor meant to capture a common source of dynamics in X_t. I assume $E(u_{i,t}) = 0$, $E(e_t) = 0$, and $E(u_{i,t}e_t) = 0$, and an exact factor structure, such that $E(u_{i,t}u_{j,t}) = 0$. Finally, I assume normality of $u_{i,t}$ and e_t.[6] This model structure follows closely that in Stock and Watson (1991). The model is estimated using Bayesian techniques, specifically the Gibbs Sampler outlined in Kim and Nelson (1998), which provides draws from the posterior distribution $p(Y_t^\star|X_1, X_2, \ldots, X_T)$. The estimated dynamic factor, Y_t, is then defined as the median of these posterior draws.

The approach to classification outlined in this section corresponds to the two-step approach proposed by Diebold and Rudebusch (1996), in which a dynamic factor is first estimated via a linear transition equation, and then a Markov-switching model is fit to the estimate of this dynamic factor. An alternative approach is to jointly estimate these components, by directly incorporating Markov-switching into the transition equation of Equation (5) and estimating the resulting nonlinear state-space model. I choose the two step approach here for several reasons. First, incorporating Markov-switching directly into Equation (5) provides another level of flexibility in the model whereby the regime switching can be used to fit idiosyncrasies in individual series, thus confounding classification of business cycle phases. Second, Camacho et al. (2015) show that for the four series considered here, the two step and joint estimation approaches provide very similar results for estimates of the Markov-switching state variable. Finally, the introduction of Markov-switching in quantiles complicates interpretation and estimation of the jointly estimated DFMS model consider-

[6]The model is normalized by setting the variance of $e_t = 1$.

ably. Given the small expected benefit, here I focus on the two step approach, which allows a clean view of the contribution of allowing for Markov-switching in quantiles for business cycle classification.

3 Vintage Data Set

This section describes the dataset used to produce the estimated dynamic factor Y_t, which is then subsequently used in estimation of the MSM and MSQ(τ) models. Because I am interested not only in historical classification of business cycle phases, but also in real-time classification, I collect a vintage dataset. This dataset is an updated version of that in Chauvet and Piger (2008) and covers four monthly coincident indicators that have been the focus of much of the U.S. business cycle dating literature. These series are the growth rate of non-farm payroll employment (PAYEMS), the growth rate of the industrial production index (IP), the growth rate of real personal income excluding current transfer receipts (W875RX1), and the growth rate of real manufacturing and trade industries sales (CMRMTSPL), where the Federal Reserve Economic Database (FRED) data code is listed in parentheses.

We consider vintages of these data as they were available at the end of each month, beginning with December 31, 1976 and updating each month to the final vintage, September 30, 2025. At each vintage, the dataset used is a balanced panel that consists of the longest overlapping sample available for all series at that vintage.[7] The first vintage holds a sample from February 1959 through October 1976. Beginning with the February 28, 1996 vintage, the sample's beginning date moves to February 1967 due to a shortening of the real manufacturing and trade industries sales series.[8] At the final vintage the sample extends from February 1967 through July 2025. This final vintage dataset will form the basis for the historical classification analysis discussed in Section 4, while earlier vintages will be used for

[7]At any particular vintage the end date of the sample will differ across series due to differential reporting lags. The beginning date of the sample can also differ, as series have different historical coverage.

[8]This change to the beginning date of the real manufacturing and trade industries sales dataset was caused by the switch from SIC to NAICS-based industry classifications.

the real-time classification experiment reported in Section 5.

I collect this vintage dataset beginning with the dataset used by Chauvet and Piger (2008), which was collected partly from the Federal Reserve Bank of Philadelphia real-time data archive described by Croushore and Stark (2001), and partly from past releases by government statistical agencies. This dataset was then extended using data releases from the Bureau of Economic Analysis and the Federal Reserve Bank of St. Louis ALFRED database.

4 Historical Business Cycle Classification

This section presents results on the historical accuracy of the MSM and MSQ(τ) models as unsupervised classifiers of U.S. data into NBER-defined business cycle phases. The results are based on final vintage data, which contains a sample from February 1967 through July 2025. The dynamic factor model in Equation (5) is first estimated as described in Section 2, and Y_t is constructed as the posterior median of $Y_t^\star$. Y_t is then used as the data to estimate the MSM and MSQ(τ) models, where τ takes values from 0.2 to 0.8 in increments of 0.05. For each model, I construct the posterior mean of S_t^h, $h \in \{m, \tau\}$, which has the interpretation of $\Pr\left(S_t^h | Y_1, Y_2, \ldots, Y_T\right)$. This posterior mean will serve as the unsupervised classifier for business cycle phases. Going forward I refer to this classifier as $\widehat{S}_t^h$.

Before discussing the results, I first review the metrics used to evaluate the classification performance of the different models. Consider a threshold, $0 \leq c \leq 1$, such that we classify month t as a recession month if $\widehat{S}_t^h \geq c$. The True Positive Rate ($TPR\,(c)$) is the proportion of NBER recession months correctly classified by $\widehat{S}_t^h$, and the False Positive Rate ($FPR\,(c)$) is the proportion of NBER expansion months incorrectly classified by $\widehat{S}_t^h$ as recession months. There is a clear tradeoff between $TPR\,(c)$ and $FPR\,(c)$, as $TPR\,(c)$ can be driven to one as c falls and $FPR\,(c)$ can be driven to zero as c increases. The ROC curve plots $TPR\,(c)$ against $FPR\,(c)$ for $0 \leq c \leq 1$, and is a visual summary of this tradeoff. The Area under the ROC Curve (AUC) measures the total area under the ROC curve, and is a common scalar summary classification metric for a binary classifier. The AUC has the intuitive interpretation as the probability that $\widehat{S}_t^h$ will be higher for a randomly chosen recession month than it would be for a randomly chosen expansion month. An AUC of 1.0 represents a perfect classifier, while an AUC of 0.5 represents a classifier no better than random guessing. I also consider the Precision of the classifier, ($PREC\,(c)$), which is the proportion of all months labeled by $\widehat{S}_t^h$ as a recession that were actually NBER recession months. Finally, I report the $F1$-Score, $F1\,(c)$, which is the geometric mean of $TPR\,(c)$ and $PREC\,(c)$, and is a common metric to balance the tradeoffs that exist between $TPR\,(c)$ and $PREC\,(c)$ for alternative values of c. The $F1$-Score is bounded between 0 and 1, with higher values indicating better classification performance.

The AUC statistic does not depend on the value of c, instead summarizing classification performance across a range of c. The other statistics are constructed for a particular value of c, and thus require a value of c be chosen, which I label $c^\star$. For the classifier from each model, I set the value of $c^\star$ to the optimal value of c calculated as the maximum of Youden's Index:

$$c^\star = \max_c \left(TPR\,(c) - FPR\,(c)\right) \tag{6}$$

Table 2 reports the classification metrics constructed for alternative models over the period from February 1967 through July 2025. The first row of Table 2 shows that the MSM model has very poor classification performance. The AUC is 0.51, indicating that the average classification performance is only slightly better than what would be expected from random guessing. $TPR(c^\star)$ is also very low, with the classifier identifying only 2% of recession months correctly. On the other hand, $FPR(c^\star)$ and $PREC(c^\star)$ are both perfect, indicating that the MSM model never produces a false positive. Finally, $F1(c^\star)$ is very low (near zero), indicating very poor overall classification performance. Figure 3 plots the value of $\widehat{S}_t^m$ over this sample period and demonstrates the reason for this poor classification performance. Over the sample, $\widehat{S}_t^m$ is essentially zero except for the two months of the NBER pandemic recession. This leads to low values of TPR and FPR, high Precision, and overall poor average classification performance.

The remaining rows of Table 2 show the results for the MSQ(τ) models. Scanning down the columns, it is clear that the MSQ(τ) models have much better overall classification performance than the MSM model. The AUC is well above 0.9 for all values of τ, and $F1(c^\star)$ is above 0.75. $TPR(c^\star)$ is above 0.8 for all values of τ, while $FPR(c^\star)$ never rises above 0.05. $PREC(c^\star)$ is above 0.6 in all cases. Evidently, the MSQ(τ) models are far more robust to the inclusion of pandemic affected data than the MSM model.

An interesting question is whether certain quantiles provide better historical classification performance than others. Judging from the summary evidence provided by the AUC and $F1$-Score, the worst performing quantiles are some of the most extreme considered, τ =0.2, 0.25 and 0.8. Over the entire range of τ, the best performing quantile across every classification metric is the 30^{th} quantile. This quantile particularly separates itself from other quantiles in $FPR(c^\star)$ and $PREC(c^\star)$, which leads to an $F1$-Score that is well above the other quantiles.

To gain more insight into the strong classification performance of the MSQ(τ) models, Figure 4 plots the value of $\widehat{S}_t^\tau$ for $\tau = 0.3$ over this sample period. For all NBER recessions in the sample, $\widehat{S}_t^\tau$ rises markedly near the beginning of the recession and falls markedly near the end. The classifier is also near zero during most NBER expansion months. To see this in terms of peak and trough dates, Table 3 shows the peak and trough dates implied by this classifier, where a peak date is defined as a month t where $\widehat{S}_t^\tau < c^\star$ and $\widehat{S}_{t+1}^\tau \geq c^\star$, and a trough date is defined as a month t where $\widehat{S}_t^\tau \geq c^\star$ and $\widehat{S}_{t+1}^\tau < c^\star$. Table 3 also shows the NBER peak and trough dates for comparison. The MSQ($\tau = 0.3$) classifier performs very well at matching the NBER peak and trough dates. Of the 16 peak and trough dates in the sample, the classifier matches the NBER peak or trough date exactly in 11 cases, and is within 3 months in 3 other cases. The largest discrepancy is six months. Also, there are no false peaks or troughs classified by the model as every identified peak and trough date corresponds reasonably closely to an NBER peak or trough.

The results above demonstrate that in a dataset that includes data from the pandemic recession, the MSM model is a very poor classifier of NBER recession months. Meanwhile, the classification performance of the MSQ(τ) models appear much more robust to this outlier recession. While this robustness is comforting, a reasonable question is whether the MSM model's sharp deterioration in performance would have been expected if the model was faced with an outlier recession that was less extreme. In other words, over what range for the severity of outlier regimes would the robustness properties of the MSQ(τ) model be valued?

For the MSM model, regime-switching is captured in the mean of the estimated dynamic factor Y_t. Thus, the key factor in whether a recession represents an outlier recession is whether the mean of Y_t in that recession is abnormally different from the non-outlier recessions in the sample. To measure this I compute the following statistic:

$$\frac{\bar{Y}_{\text{recession}_j}}{\bar{Y}_{\text{recession}_{\neq 2020}}} \tag{7}$$

where $\bar{Y}_{\text{recession}_j}$ is the average value of Y_t over the months of recession j and $\bar{Y}_{\text{recession}_{\neq 2020}}$ is the average value of Y_t over all recessions excluding the 2020 pandemic recession. Table 4 presents this statistic for each recession from February 1967-July 2025, and provides two important takeaways. First, for recessions from 1980-2019, there was not a recession that stands out as an obvious outlier by the metric in Equation (7). The largest outliers on the positive side are the 1980 and 2008-2009 recession, for which $\bar{Y}_{\text{recession}_j}$ was 1.35 times the average pre-pandemic recession, and on the negative side was the 2001 recession, for which $\bar{Y}_{\text{recession}_j}$ was 0.65 times the average. Thus, over the pre-pandemic sample period, there was relatively mild discrepancy across recessions in the average value of Y_t observed during recessions. The second takeaway is that the 2020 pandemic recession is a truly extreme outlier, being nearly 12 times more severe than the average pre-pandemic recession.

Table (4) demonstrates that the MSM model, which was in use in various forms relatively continuously since the publication of Hamilton (1989), was not seriously tested by outlier recessions. Then, with the pandemic recession, it was tested with an extraordinary, and perhaps isolated, outlier. A reasonable question is then to ask how the MSM model would likely perform in the face of a less severe outlier recession. If the MSM model's performance would only deteriorate when a pandemic level recession occurs, then one could argue that simply ignoring the pandemic recession in estimation and using the MSM model going forward would be a reasonable strategy. However, if the MSM's model ability to classify would deteriorate materially with a much smaller outlier recession, then the value of a robust classifier such as MSQ(τ) is more apparent.

To investigate this question I conduct an experiment in which data is simulated from the MSM model, using the posterior median parameters estimated from a pre-pandemic sample running from February 1967 through December 2019. Each simulation includes generation of the regime indicator, S_t^m, which defines the true recessions in the simulated data.[9] The data is simulated for an identical size dataset as our final vintage data, from February 1967 through July 2025. In each simulation, one of the recessions generated in that simulation is scaled upward to be an outlier recession as defined by the metric in Equation (7). I consider outlier recessions from 1 times more severe, which corresponds to an absence of outliers, up to 10 times more severe, which is just short of the pandemic recession. The MSM and MSQ(τ) models are estimated for each simulated series, expansion and recession months are classified, and the $F1$-Score computed. The median, along with 20^{th} and 80^{th} percentiles of the $F1$-Score across 1000 simulations are reported in Table 5. For the MSQ(τ) models I report results for three representative values of $\tau \in \{0.25, 0.5, 0.75\}$. Table 6 shows the results for the AUC, and provides similar conclusions.

The first row of Table 5 shows the case where there is no outlier recession. Given that data is simulated from the MSM model, we would expect the MSM model to perform very well in this setting. The $F1$-Score is very high (0.96) in this case. Interestingly, the MSQ(τ)

[9]We discard all simulations that generate one-month recessions.

models have very similar performance, demonstrating that there is not a significant cost to using the MSQ(τ) models in a setting where robustness is not important. The second row of Table 5 shows that all the models are robust to a single outlier recession that is 2 times more severe than the average recession. The $F1$-Score in this case does not fall materially for any of the models. When we move to the case of a recession that is 4 times the average, we see a decline in the performance of the MSM model. The $F1$-Score drops marginally, to 0.9. But the 20^{th} percentile simulation drops to 0.3, indicating that in at least 20% of simulations the MSM displayed very poor classification performance. Meanwhile the performance of the MSQ(τ) models are largely unaffected. Scanning through the remaining rows, the MSM model performs very poorly for outlier recessions that are 6, 8, or 10 times the average, with median $F1$-Score values that are below 0.5. Again, the MSQ(τ) models perform well across all of these cases, demonstrating impressive robustness in classifying recessions in the presence of outlier recessions of varying severity. Overall, Table 5 demonstrates that the performance decline of the MSM model would be expected with an outlier recession far less severe than the pandemic recession, which further validates the importance of using an approach that is robust to such events.

5 Real-Time Business Cycle Classification

The results of the previous section demonstrate that the MSQ(τ) model is capable of providing an accurate unsupervised classification of U.S economic data into historical NBER expansion and recession regimes. This section goes further and asks how the MSQ(τ) would have performed as a real-time classifier of new business cycle turning points over the past 50 years.

To investigate the real-time performance of the MSQ(τ) models, I perform a similar real-time classification experiment to that considered in Chauvet and Piger (2008). Consider an analyst who is using the MSQ(τ) models to identify new business cycle turning points in real time, updating their analysis at the end of each month beginning with the first vintage in the dataset, December 31, 1976, and ending with the last vintage, September 30, 2025. The analyst uses a balanced panel consisting of the largest overlapping sample available for the four series described in Section 3 to estimate the dynamic factor model in (5) and the MSQ(τ) models in (3). Label the ending month of this balanced panel as T. The analyst then identifies new business cycle turning points using the rules taken from Chauvet and Piger (2008), which I detail here for a business cycle peak: Suppose the most recently identified business cycle turning point was a trough, meaning the analyst is now searching for a new business cycle peak. To identify a new business cycle peak, the analyst requires that $\widehat{S}^{\tau}_{T-j} \geq 0.8, j = 0, 1, 2$. Once the existence of a new peak is established, the timing of this peak is set to month $T - j$, where j is the smallest possible value of j such that $\widehat{S}^{\tau}_{T-j} < 0.5$ and $\widehat{S}^{\tau}_{T-j+1} \geq 0.5$. An analogous procedure, with the 80% threshold replaced by 20%, is used to identify new business cycle troughs. I consider two values of τ. The first is the lower quartile ($\tau = 0.25$), which is both a representative below median quantile and is close to the value of $\tau = 0.3$ that provided the best performance in the classification of historical business cycle phases. The second is the median ($\tau = 0.50$), which is perhaps the most obvious robust alternative to the MSM model.

Table 7 shows the real time performance of the MSQ(τ) models for identifying new business cycle peaks. The first two columns show the NBER peak date as well as the date at which the NBER peak date was announced by the NBER's Business Cycle Dating Committee. The third and fourth columns show the first identified peak date by the MSQ($\tau = 0.25$) model and the date that this peak date was identified. The final columns present this information for the MSQ($\tau = 0.50$) model.

From the table, both MSQ(τ) models identify the peak of all six recessions in the real-time sample. That is, there are no false negatives produced by the models. The MSQ(τ) models also establish these dates with reasonable accuracy. The MSQ($\tau = 0.25$) displays the closest correspondence with the NBER peak dates, with established peak dates within 1.3 months of the corresponding NBER peak on average. The MSQ($\tau = 0.50$) is slightly less accurate, being within 3 months of the NBER date on average. Both MSQ(τ) models also establish peaks more quickly on average than the NBER. For three recessions the MSQ(τ) models would have announced the peak inside of the same month as the NBER announcement. For the other three recessions, the MSQ(τ) models would have announced the peak between 4-8 months earlier than the NBER. On average the MSQ($\tau = 0.25$) model would have led the NBER by 73 days, and the MSQ($\tau = 0.50$) model by 68 days.

Table 8 shows the real-time performance of the MSQ(τ) models for identifying new business cycle troughs. Both MSQ(τ) models identify the trough of all six recessions in the real-time evaluation sample with exceptional accuracy. Indeed, both model produce an initial peak date that is exactly equal to the NBER date for 5 of the 6 recessions in the sample, and is only one month different for the remaining recession. The MSQ(τ) models also establishes these trough dates earlier than the NBER for all six business cycle troughs, with a lead time of between 39 and 449 days.

The MSQ(τ) models each produce two false recessions in real time, which are detailed in the lower panel of Tables 7 and 8. Both of these episodes correspond to the time periods often referred to as jobless recoveries, one following the 1990-1991 recession and the other following the 2001 recession. These false recessions are also later revised away by the MSQ(τ) models, as they do not appear in the final vintage chronology of business cycle dates for either model.

This real-time classification experiment was dominated by a sample period that did not contain outlier recessions. An interesting remaining question is how the MSQ(τ) models would be expected to perform at identifying new business cycle phases going forward, now that the sample has been impacted by the pandemic recession. To investigate this, I conduct a simulation experiment where simulated future vintages of data are generated, and the MSQ(τ) models are applied to these simulated vintages in real time.

To simulate future vintages, I first estimate the MSM model on a pre-pandemic dataset that extends from February 1967 through December 2019. Using the posterior median parameter estimates from this estimation, I then generate a value for Y_{T+1} from the MSM model and append this to the final vintage values of $Y_1, Y_2, \ldots, Y_T$. I then continue the search for new business cycle turning points using the MSQ(τ) models on this new vintage of data. This is then repeated through period $T + G$, thus creating G total vintages of data.

I conduct this real-time simulation experiment for 1000 simulations of $G = 120$ future monthly vintages. In other words, each simulation depicts an analyst attempting to identify business cycle turning points for ten years following the end of our final vintage of data. Table 9 presents these results. On average, there were 1.6 peaks generated per ten-year simulation. Across these simulations, the MSQ($\tau = 0.25$) model identified 1.4 of these peaks (88%) in real time on average. The MSQ($\tau = 0.50$) model identified 1.3 peaks (81%). This performance comes with a small number of false peaks, particularly when $\tau = 0.25$. In this case, the model produces 0.3 false peaks per ten-year period on average, while this increases to 0.6 false peaks when $\tau = 0.50$.

6 Measuring Business Cycles in the United Kingdom

The preceding analysis has focused on U.S. data. In this section I provide evidence on the effect that the global pandemic had on the ability of Markov-switching models to classify recessions in the United Kingdom, and whether MSQ models provide a robust alternative. Specifically, I consider the MSM and MSQ(τ) models applied to U.K. quarterly real GDP growth. That is, Y_t in Equations (1) and (3) is U.K. quarterly real GDP growth.[10]

Figure 5 shows these results. The top panel plots $\widehat{S}_t^m$ for the MSM model estimated on a pre-pandemic sample from 1955:Q2-2019:Q4 (top left panel) and on a sample that includes the pandemic from 1955:Q2-2025:Q1 (top right panel). The results are similar to those for the United States. The MSM model as a classifier of business cycle regimes is not robust to the inclusion of pandemic-era data, and the effect of including this data is to cause the

[10] The U.K. real GDP series is obtained from the Federal Reserve Economic Database, with data code NGDPRSAXDCGBQ

model to define only a single low-growth regime corresponding to a pandemic recession of similar timing to that observed in the United States.

The bottom panel of Figure 5 shows the analogous results for the MSQ(τ) models. Given the good performance of the model for $\tau = 0.3$ in the United States, I consider that model here. Comparing the bottom left and bottom right panels, the estimated value of $\widehat{S}_t^\tau$ when estimated only on pre-pandemic data, or when estimated on data including the pandemic recession, is similar. Thus, as was the case for the United States, classification of U.K. business cycle phases via MSQ models is robust to the inclusion of pandemic-affected data.

7 Conclusion

This paper has considered the problem of identifying business cycle phases, both retrospectively and in real time, when faced with a dataset that contains outlier recessions, such as the recession associated with the Covid-19 pandemic. Standard Markov-switching models, which incorporate Markov-switching into the mean of a process, display a sharp deterioration in their ability to classify business cycle phases when the pandemic recession is included in the estimation sample. Further, simulations reveal that these models would have experienced similarly deteriorated performance if faced with an outlier recession far less severe than the pandemic recession, increasing the relevance and importance of developing robust methods.

I have proposed Markov-switching quantile (MSQ) models as a potentially robust alternative. I find that MSQ models provide an accurate unsupervised classification of historical U.S. recession and expansion phases when applied to an estimated dynamic factor of U.S. macroeconomic data, and this accuracy is robust to inclusion of data from the pandemic. I also consider the performance of the MSQ model for identifying new business cycle turning points in real time by extending the real-time analysis of Chauvet and Piger (2008). The MSQ model provides timely and accurate classification of new U.S. business cycle peaks and troughs over the past 50 years. Further, through simulations, I show that the MSQ model is likely to continue to provide accurate classification in future data. Finally, I show that robust performance of the MSQ model extends to business cycle classification in the United Kingdom.

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A Appendix: Gibbs Sampler for Markov-Switching Quantile Model

This appendix lays out the Gibbs Sampler used for estimation of the Markov-switching quantile (MSQ(τ)) models described in Section 2.

A-1 Markov-Switching Quantile Model

Again, the MSQ(τ) model is described by the following equations:

$$\begin{aligned} Y_t &= \alpha^{\tau}_{S^{\tau}_t} + \sigma^{\tau}\omega^{\tau}_t, \\ S^{\tau}_t &\in \{0,1\}, \\ \Pr(S^{\tau}_t = i | S^{\tau}_{t-1} = j) &= P^{\tau}_{ij}, \\ \omega^{\tau}_t &\sim AL\,(0,1,\tau) \end{aligned} \tag{A-1}$$

A-2 Mixture Representation

Let $z_t \geq 0$ have an independent standard exponential density:

$$z_t \sim \text{i.i.d. } Exp(1)$$

so that $f(z_t) = e^{-z_t}$. In this parameterization, $E(z_t) = 1$. Also, let $v_t \sim$ i.i.d. $N(0,1)$. As discussed in Kozumi and Kobayashi (2011), and shown in Kotz et. al (2001), we can represent $\omega^{\tau}_t \sim$ i.i.d. $AL(0,1,\tau)$ as the following mixture:

$$\omega^{\tau}_t = \theta z_t + \pi\sqrt{z_t}v_t$$

where:

$$\theta = \frac{1-2\tau}{\tau(1-\tau)}$$
$$\pi^2 = \frac{2}{\tau(1-\tau)}$$

The MSQ(τ) model be equivalently written in terms of this mixture as:

$$Y_t = \alpha^\tau_{S^\tau_t} + \sigma^\tau \theta z_t + \sigma^\tau \pi \sqrt{z_t} v_t.$$

Equivalently, but more convenient for the development of a Gibbs Sampler, we can write the model as:

$$y_t = \alpha^\tau_{S^\tau_t} + \theta z^\star_t + \pi \sqrt{\sigma^\tau z^\star_t} v_t,$$

where $z^\star_t = \sigma^\tau z_t$. This will imply that $z^\star_t \sim Exp\left(\sigma^\tau\right)$, where σ^τ is the scale parameter of the exponential density.

A-3 Priors

The prior densities for the parameters of the MSQ(τ) models are given in Table (1). Here I briefly review the parametric families used to set these prior densities. I assume prior independence across parameter blocks:

$$p\left(\alpha^\tau, \sigma^\tau, P^\tau\right) = p\left(\alpha^\tau\right) p\left(\sigma^\tau\right) p\left(P^\tau\right)$$

where $\alpha^\tau = \left(\alpha^\tau_0, \alpha^\tau_1\right)'$ and $P^\tau = \left(P^\tau_{00}, P^\tau_{11}\right)'$. The regime-dependent means are described with a Gaussian prior:

$$\alpha^\tau \sim N(\mu, V)$$

The scale parameter has the standard improper prior:

$$p\left(\sigma^\tau\right) = \left(\sigma^\tau\right)^{-1}$$

Finally, the transition probabilities have independent Beta prior densities:

$$p(P^\tau) = p(P_{00}^\tau)\, p(P_{11}^\tau)$$

$$P_{00}^\tau \sim Beta(B_{00}, B_{01})$$

$$P_{11}^\tau \sim Beta(B_{11}, B_{10})$$

A-4 Conditional Posteriors

Define the data available for estimation as $Y = (Y_1, Y_2, \ldots, Y_T)$. We are interested in sampling from the posterior distribution for the model parameters:

$$p(\alpha^\tau, \sigma^\tau, P^\tau | Y)$$

as well as the posterior distribution for the Markov-switching state variable:

$$\Pr(S^\tau | Y)$$

To sample from these posterior distributions via the Gibbs Sampler, I will sample from the full set of conditional posterior densities. Note that $\Pr(S^\tau | Y)$ is of central interest in its own right, as this will be the object that informs our classification. However, including S^τ into the objects drawn plays a secondary purpose, as being able to condition on S^τ will greatly simplify the conditional posterior distribution for the model parameters. This is an example of data augmentation (Tanner and Wong (1987)). As another instance of data augmentation, I additionally include $z^\star$ in the sampler, which additionally simplifies the derivation of the parameter conditional posterior densities. Specifically, the Gibbs Sampler will iterate through draws from the following densities:

(1) Parameter Block

$$p(\alpha^\tau | Y, S^\tau, z^\star, \sigma^\tau, P^\tau)$$

$$p(\sigma^\tau | Y, S^\tau, z^\star, \alpha^\tau, P^\tau)$$

$$p\left(P^{\tau}|Y, S^{\tau}, z^{\star}, \alpha^{\tau}, \sigma^{\tau}\right)$$

(2) Data Augmentation Block

$$\Pr\left(S^{\tau}|Y, z^{\star}, \alpha^{\tau}, \sigma^{\tau}, P^{\tau}\right)$$

$$\Pr\left(z^{\star}|Y, S^{\tau}, \alpha^{\tau}, \sigma^{\tau}, P^{\tau}\right)$$

Details for drawing each of these densities is provided below.

A-4.1 $\alpha^{\tau}|\mathbf{Y}, \mathbf{S}^{\tau}, \mathbf{z}^{\star}, \sigma^{\tau}, \mathbf{P}^{\tau}$

Define $W_t = [(1 - S_t) \;\; S_t \;\;]'$. Then:

$$\alpha^{\tau}|Y, S^{\tau}, z^{\star}, \sigma^{\tau}, P^{\tau} \sim N\left(\bar{\mu}, \bar{V}\right)$$

where:

$$\bar{V} = \left(\sum_{t=1}^{T}\left(W_t W_t'/\tau^2\sigma^{\tau}z_t^{\star}\right) + V^{-1}\right)^{-1}$$
$$\bar{\mu} = \bar{V}\left(\sum_{t=1}^{T}\left(W_t^{\star}\left(Y_t - \theta z_t^{\star}\right)/\tau^2\sigma^{\tau}z_t^{\star}\right) + V^{-1}\mu\right)$$

A-4.2 $\sigma^{\tau}|\mathbf{Y}, \mathbf{S}^{\tau}, \mathbf{z}^{\star}, \alpha^{\tau}, \mathbf{P}^{\tau}$

$$\sigma^{\tau}|Y, S^{\tau}, z^{\star}, \alpha^{\tau}, P^{\tau} \sim IG\left(\bar{\alpha}, \bar{\gamma}\right)$$

where IG is the inverted Gamma density and:

$$\bar{\alpha} = 3T/2 + \alpha$$
$$\bar{\gamma} = \gamma + \frac{1}{2}\sum_{t=1}^{T}\frac{\left(y_t - w_t'\kappa_{p,S_t} - \theta z_t^{\star}\right)^2}{\tau^2 z_t^{\star}} + \sum_{t=1}^{T} z_t^{\star}$$

A-4.3 $\mathbf{P}^{\tau}|\mathbf{Y}, \mathbf{S}^{\tau}, \mathbf{z}^{\star}, \alpha^{\tau}, \sigma^{\tau}$

It is straightforward to show that:

$$f\left(P^{\tau}|Y,S^{\tau},z^{\star},\alpha^{\tau},\sigma^{\tau}\right)$$
$$\propto \Pr\left(S_1|P\right)\left(P_{00}^{\tau}\right)^{N_{00}+B_{0,0}-1}\left(1-P_{00}^{\tau}\right)^{N_{01}+B_{0,1}-1}\left(P_{11}^{\tau}\right)^{N_{11}+B_{1,0}-1}\left(1-P_{11}^{\tau}\right)^{N_{10}+B_{1,1}-1}$$

where $N_{ij}=\sum_{t=2}^{T} I(S_{t-1}=i)I(S_t=j)$.

The term $\Pr(S_1|P)$ makes this density non-standard, and it would require a Metropolis-Hastings step to sample it directly. However, the usual practice is to ignore this density of the initial observation (set it equal to one) which is in the spirit of conditional estimation often used with autoregressive processes. This will have little effect on results for a reasonably large sample size. With this assumption, we have:

$$P_{00}^{\tau}|Y,S^{\tau},z^{\star},\alpha^{\tau},\sigma^{\tau},P^{\tau}\sim Beta(N_{00}+B_{0,0},N_{01}+B_{0,1})$$
$$P_{11}^{\tau}|Y,S^{\tau},z^{\star},\alpha^{\tau},\sigma^{\tau},P^{\tau}\sim Beta(N_{10}+B_{1,0},N_{11}+B_{1,1})$$

A-4.4 $\mathbf{S}^{\tau}|\mathbf{Y},\mathbf{z}^{\star},\alpha^{\tau},\sigma^{\tau},\mathbf{P}^{\tau}$

Using the law of total probability, and recognizing the Markov property of S^{τ}, this density can be factored as follows:

$$\Pr\left(S^{\tau}|Y,z^{\star},\alpha^{\tau},\sigma^{\tau},P^{\tau}\right)=\Pr\left(S_T^{\tau}|\widetilde{Y}_T,\widetilde{z}_T^{\star},\alpha^{\tau},\sigma^{\tau},P^{\tau}\right)\prod_{t=1}^{T-1}\Pr\left(S_t^{\tau}|S_{t+1}^{\tau},\widetilde{Y}_T,\widetilde{z}_T^{\star},\alpha^{\tau},\sigma^{\tau},P^{\tau}\right)$$

where $\widetilde{Y}_t$ and $\widetilde{z}_t^{\star}$ contain the elements of Y and $z^{\star}$ from periods 1 through t. This factorization gives us an approach to draw from $\Pr\left(S^{\tau}|Y,z^{\star},\alpha^{\tau},\sigma^{\tau},P^{\tau}\right)$ by drawing recursively, beginning with $\Pr\left(S_T^{\tau}|\widetilde{Y}_T,\widetilde{z}_T^{\star},\alpha^{\tau},\sigma^{\tau},P^{\tau}\right)$ and moving backwards from $t=T,T-1,\ldots,1$. To implement this procedure, first note that:

$$\Pr\left(S_t|S_{t+1},\widetilde{y}_t,\widetilde{x}_t,\beta_p,\widetilde{z}_t^{\star},\sigma,P\right)\propto\Pr\left(S_{t+1}|S_t\right)\Pr\left(S_t|\widetilde{y}_t,\widetilde{x}_t,\beta_p,\widetilde{z}_t^{\star},\sigma,P\right)$$

Second, we can compute the "filtered" state probabilties, $\Pr\left(S_t^{\tau}=i|\widetilde{Y}_t,\widetilde{z}_t^{\star},\alpha^{\tau},\sigma^{\tau},P^{\tau}\right)$, $i=$

$0, 1$, for $t = 1, \ldots, T$ using the recursive filter in Hamilton (1989). Taken together this provides sufficient information to draw S_T^τ from $\Pr\left(S_T^\tau | \widetilde{Y}_T, \widetilde{z}_T^\star, \alpha^\tau, \sigma^\tau, P^\tau\right)$ and then S_t^τ from $\Pr\left(S_t^\tau | S_{t+t}^\tau, \widetilde{Y}_T, \widetilde{z}_T^\star, \alpha^\tau, \sigma^\tau, P^\tau\right)$ for $t = T-1, T-2, \ldots, 1$.

A-4.5 $\mathbf{z}^\star | \mathbf{Y}, \mathbf{S}^\tau, \alpha^\tau, \sigma^\tau, \mathbf{P}^\tau$

Due to the independence of $z_t^\star$, we have:

$$f\left(z^\star | Y, S^\tau, \alpha^\tau, \sigma^\tau, P^\tau\right) = \prod_{t=1}^{T} f\left(z_t^\star | Y_t, S_t, \alpha^\tau, \sigma^\tau, P^\tau\right)$$

As is shown in Khare and Hobert (2012), $z_t^\star | Y_t, S_t, \alpha^\tau, \sigma^\tau, P^\tau$ has reciprocal inverse-Gaussian density:

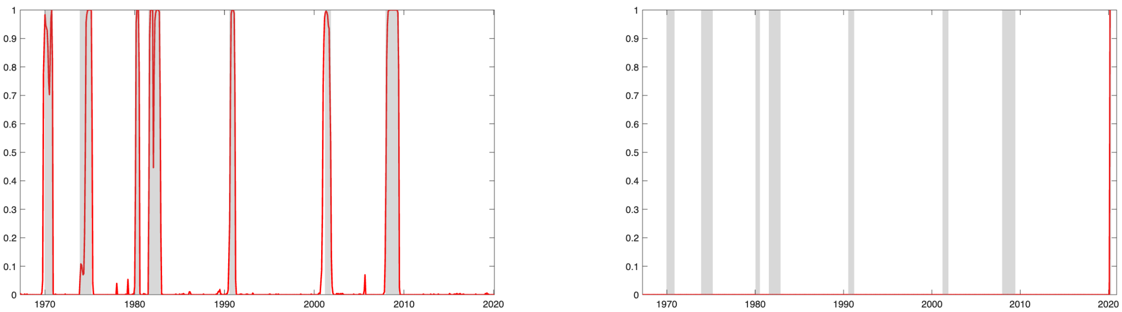
$$f\left(z_t^\star | Y_t, S_t, \alpha^\tau, \sigma^\tau, P^\tau\right) = \sqrt{\frac{\lambda}{2\pi x}} \exp\left(\frac{-\lambda\left(1 - mx\right)^2}{2m^2 x}\right)$$

where:

$$\lambda = \frac{\theta^2 + 2\pi^2}{\pi^2 \sigma}$$

$$m = \frac{\sqrt{\theta^2 + 2\pi^2}}{|y_t - \alpha_{S_t^\tau}^\tau|}$$

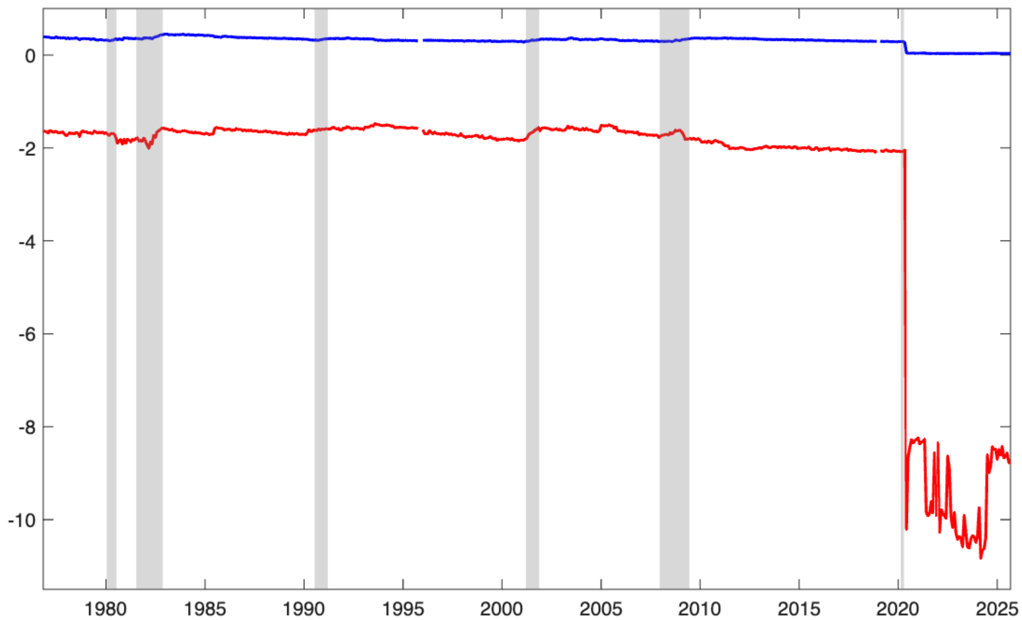
Figure 1
Estimated U.S. Recession Probability from Markov-Switching Mean Model



Feb. 1967-Feb.2020 **Feb. 1967-Apr.2020**

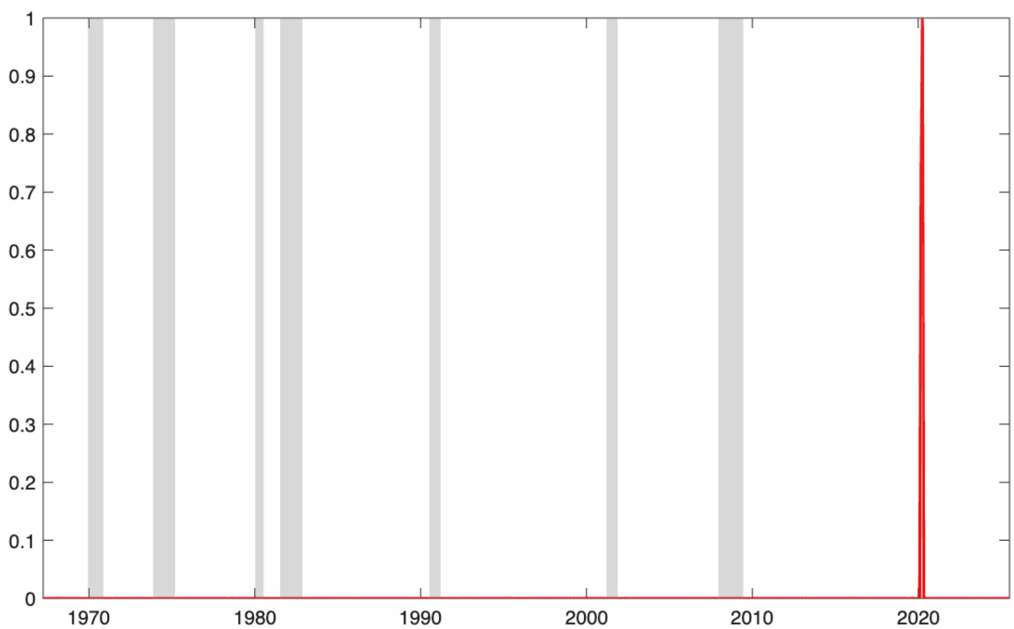
Notes: Monthly probability of a U.S. recession based on the MSM model in Equation (1) applied to a common dynamic factor of the growth rates of payroll employment, industrial production, real personal income excluding transfer payments and real manufacturing and trade industries sales. Estimation sample is February 1967-February 2020 (left panel) and February 1967-April 2020 (right panel). Shading indicates NBER-defined recession months.

Figure 2
Expanding Window Estimation of Mean of Dynamic Factor in Low Mean Regime and High Mean Regime



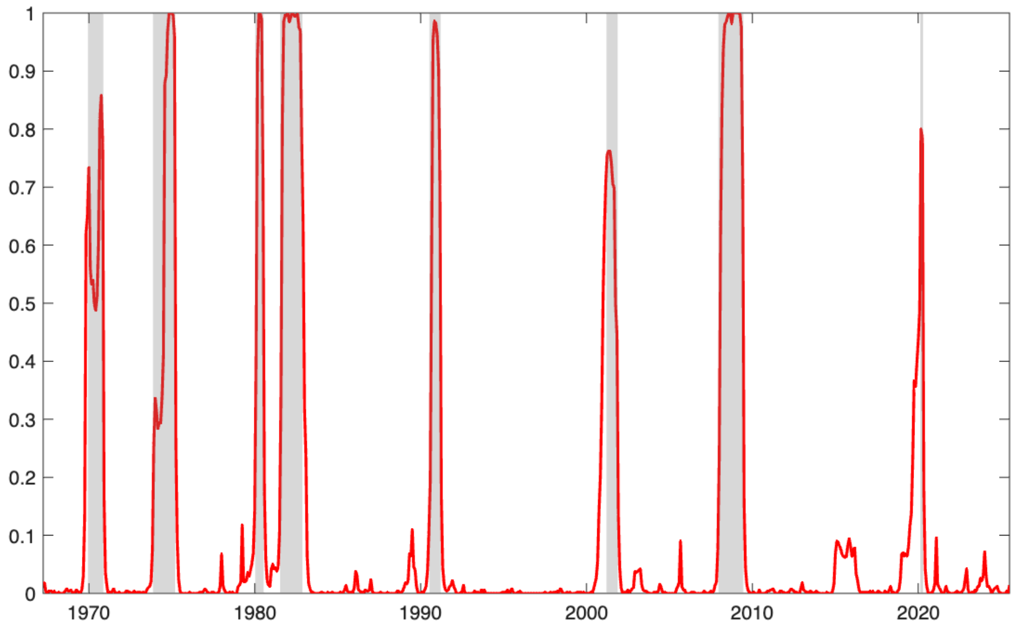
Notes: Expanding window estimates of regime-dependent means based on the MSM model in Equation (1) applied to a common dynamic factor of the growth rates of payroll employment, industrial production, real personal income excluding transfer payments and real manufacturing and trade industries sales. The first estimation sample is February 1967-September 1976 and the final estimation sample is February 1967-July 2025. Shading indicates NBER-defined recession months.

Figure 3
Estimated U.S. Recession Probability from Markov-Switching Mean Model
February 1967-July 2025



Notes: Monthly probability of a U.S. recession based on the MSM model in Equation (1) applied to a common dynamic factor of the growth rates of payroll employment, industrial production, real personal income excluding transfer payments and real manufacturing and trade industries sales. Estimation sample is February 1967-July 2025. Shading indicates NBER-defined recession months.

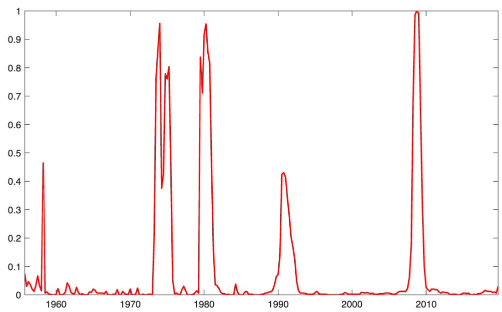
Figure 4
Estimated U.S. Recession Probability from Markov-Switching Quantile Model
February 1967-July 2025



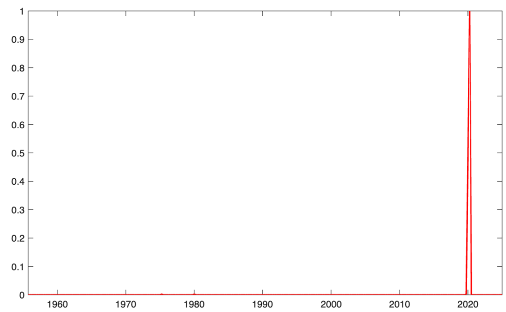
Notes: Monthly probability of a U.S. recession based on the MSQ(0.30) model in Equation (3) applied to a common dynamic factor of the growth rates of payroll employment, industrial production, real personal income excluding transfer payments and real manufacturing and trade industry sales. Estimation sample is February 1967-July 2025. Shading indicates NBER-defined recession months.

Figure 5
Estimated U.K. Recession Probability

MSM Model

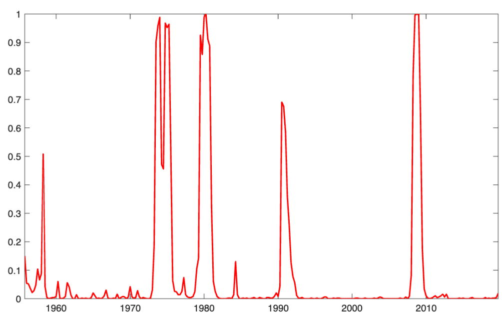


1955:Q2-2019:Q4

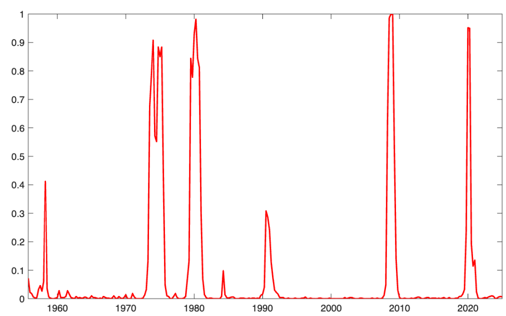


1955:Q2-2025:Q1

MSQ(0.30) Model



1955:Q2-2019:Q4



1955:Q2-2025:Q1

Notes: Quarterly probability of a U.K. recession based on the MSM model in Equation (1) (top panel) and MSQ(0.30) model in Equation (3) (bottom panel). Both models are estimated on U.K. quarterly real GDP growth over 1955:Q2-2019:Q4 (left panels) and 1955:Q2-2025:Q1 (right panels).

Table 1
Prior Densities for MSM and MSQ(τ) Model Parameters

Parameter	**Prior**
$\alpha_0^m,\ \alpha_0^\tau$	$N\,(0.5, 1.0)$
$\alpha_1^m,\ \alpha_1^\tau$	$N\,(-0.5, 1.0)$
$\sigma^m,\ \sigma^\tau$	non-informative
$P_{00}^m,\ P_{00}^\tau$	$Beta\,(9.5, 0.5)$
$P_{11}^m,\ P_{11}^\tau$	$Beta\,(0.0, 1.0)$

Table 2
Metrics for Classification of NBER Expansion and Recession Phases
February 1967 - July 2025

Model	AUC	TPR	FPR	Precision	F1
MSM	0.51	0.02	0.00	1.00	0.05
MSQ(0.20)	0.94	0.87	0.06	0.67	0.76
MSQ(0.25)	0.97	0.94	0.07	0.65	0.77
MSQ(0.30)	1.00	1.00	0.03	0.83	0.91
MSQ(0.35)	0.99	0.98	0.05	0.74	0.84
MSQ(0.40)	0.99	1.00	0.07	0.65	0.79
MSQ(0.45)	0.99	1.00	0.07	0.67	0.81
MSQ(0.50)	0.98	1.00	0.06	0.69	0.82
MSQ(0.55)	0.99	1.00	0.07	0.66	0.80
MSQ(0.60)	0.99	1.00	0.08	0.63	0.78
MSQ(0.65)	0.99	0.98	0.05	0.72	0.83
MSQ(0.70)	0.98	0.98	0.06	0.68	0.80
MSQ(0.75)	0.98	0.98	0.06	0.69	0.81
MSQ(0.80)	0.95	0.93	0.10	0.56	0.70

Notes: Accuracy metrics for monthly probability of a U.S. recession based on the MSM model in Equation (1) and MSQ models in Equation (3) as classifiers of NBER-defined recessions. Metrics are defined in Section 4.

Table 3
U.S. Peak and Trough Dates Classified by MSQ(0.30) Model

NBER Peak	MSQ(0.30) Peak	NBER Trough	MSQ(0.30) Trough
Dec. 1969	Oct. 1969	Nov. 1970	Nov. 1970
Nov. 1973	Nov. 1973	Mar. 1975	Apr. 1975
Jan. 1980	Jan. 1980	Jul. 1980	Jul. 1980
Jul. 1981	Jul. 1981	Nov. 1982	Feb. 1983
Jul. 1990	Jul. 1990	Mar. 1991	Mar. 1991
Mar. 2001	Oct. 2000	Nov. 2001	Nov. 2001
Dec. 2007	Dec. 2007	Jun. 2009	Jun. 2009
Feb. 2020	Aug. 2019	Apr. 2020	Apr. 2020

Notes: Business cycle peak and trough dates from the NBER and the MSQ(0.30) model in Equation (3). MSQ(0.30) peaks (troughs) are established by the transition of the probability of recession moving from below (above) to above (below) $c^\star$, where $c^\star$ is the maximum of Youden's Index defined in Equation (6).

Table 4
U.S. Recession Severity

NBER Recession	$\bar{Y}_{\text{recession}_j}/\bar{Y}_{\text{recession}_{\neq 2020}}$
1970	0.74
1973-1975	1.13
1980	1.38
1981-1982	0.80
1990-1991	0.79
2001	0.65
2008-2009	1.35
2020	11.88

Notes: This table presents the measure of relative recession severity from Equation (7). Y_t is a common dynamic factor of the growth rates of payroll employment, industrial production, real personal income excluding transfer payments and real manufacturing and trade industries sales, estimated over February 1967-July 2025. $\bar{Y}_{\text{recession}_j}$ is the average value Y_t in recession j, and $\bar{Y}_{\text{recession}_{\neq 2020}}$ is the average value of Y_t across the non-pandemic recessions.

Table 5
Median F1-Score over Simulations

Outlier Ratio	MSM	MSQ(25)	MSQ(50)	MSQ(75)
1.0	0.96	0.96	0.96	0.94
	(0.92,0.99)	(0.91,0.98)	(0.90,0.98)	(0.88,0.97)
2.0	0.96	0.96	0.96	0.95
	(0.93,0.99)	(0.92,0.98)	(0.92,0.99)	(0.89,0.98)
4.0	0.90	0.95	0.95	0.94
	(0.36,0.97)	(0.85,0.97)	(0.89,0.98)	(0.86,0.97)
6.0	0.41	0.96	0.96	0.94
	(0.22,0.96)	(0.87,0.99)	(0.88,0.99)	(0.89,0.98)
8.0	0.28	0.94	0.95	0.94
	(0.13,0.76)	(0.83,0.98)	(0.89,0.98)	(0.88,0.97)
10.0	0.20	0.94	0.95	0.94
	(0.10,0.39)	(0.87,0.98)	(0.88,0.98)	(0.87,0.97)

Notes: This table shows the median $F1$-Score computed from the MSM model in Equation (1) and the MSQ(τ) models in Equation (3) applied to 1000 simulated datasets, where the datasets contain one outlier recession as defined by Equation (7). The 20^{th} and 80^{th} percentile across these simulations are in parenthesis.

Table 6
Median AUC over Simulations

Outlier Ratio	MSM	MSQ(25)	MSQ(50)	MSQ(75)
1.0	0.999	0.999	0.999	0.998
	(0.998,1.000)	(0.998,1.000)	(0.998,1.000)	(0.993,1.000)
2.0	0.999	0.999	0.999	0.999
	(0.998,1.000)	(0.998,1.000)	(0.998,1.000)	(0.989,1.000)
4.0	0.991	0.999	0.999	0.998
	(0.611,0.999)	(0.995,1.000)	(0.998,1.000)	(0.992,0.999)
6.0	0.628	0.999	0.999	0.999
	(0.562,0.999)	(0.996,1.000)	(0.997,1.000)	(0.993,1.000)
8.0	0.582	0.999	0.999	0.998
	(0.536,0.811)	(0.994,1.000)	(0.997,1.000)	(0.990,0.999)
10.0	0.556	0.998	0.998	0.998
	(0.527,0.621)	(0.992,1.000)	(0.995,1.000)	(0.985,0.999)

Notes: This table shows the median AUC computed from the MSM model in Equation (1) and the MSQ(τ) models in Equation (3) applied to 1000 simulated datasets, where the datasets contain one outlier recession as defined by Equation (7). The 20^{th} and 80^{th} percentile across these simulations are in parenthesis.

Table 7
Real-Time Peak Identification from NBER and MSQ(τ) Classifier

NBER		MSQ(0.25)		MSQ(0.50)	
Peak	**Release**	**Peak**	**Release**	**Peak**	**Release**
Jan. 1980	Jun. 3, 1980	Jan. 1980	Jun. 30, 1980	Jun. 1979	Jun. 30, 1980
Jul. 1981	Jan. 6, 1982	Jul. 1981	Jan. 31, 1982	Jun. 1981	Jan. 31, 1982
Jul. 1990	Apr. 25, 1991	Jun. 1990	Dec. 31, 1990	Jun. 1990	Dec. 31, 1990
Mar. 2001	Nov. 26, 2001	Sep. 2000	Mar. 31, 2001	Sep. 2000	Apr. 30, 2001
Dec. 2007	Dec. 1, 2008	Nov. 2007	Jun. 30, 2008	Oct. 2007	Jun. 30, 2008
Feb. 2020	Jun. 8, 2020	Jan. 2020	Jun. 30, 2020	Nov. 2019	Jun. 30, 2020
—	—	Sep. 1991	Mar. 31, 1992	Sep. 1991	Mar. 31, 1992
—	—	Jul. 2002	Feb. 28, 2003	Jul. 2002	Feb. 28, 2003

Notes: This table shows the business cycle peaks identified in real time by the MSQ(τ) models in Equation (3), as well as the vintage at which these peak dates could have been released. The table also contains the NBER peak dates and the dates these were released in real time. The real time identification strategy is discussed in Section 5.

Table 8
Real-Time Trough Identification from NBER and MSQ(τ) Classifier

NBER		MSQ(0.25)		MSQ(0.50)	
Trough	**Release**	**Trough**	**Release**	**Trough**	**Release**
Jul. 1980	Jul. 8, 1981	Jul. 1980	Dec. 31, 1980	Jul. 1980	Dec. 31, 1980
Nov. 1982	Jul. 8, 1983	Nov. 1982	May 31, 1983	Nov. 1982	May 31, 1983
Mar. 1991	Dec. 22, 1992	Mar. 1991	Sep 30, 1991	Mar. 1991	Sep 30, 1991
Nov. 2001	Jul. 17, 2003	Dec. 2001	May 31, 2002	Dec. 2001	Aug. 31, 2002
Jun. 2009	Sep. 20, 2010	Jun. 2009	Dec. 31, 2009	Jun. 2009	Nov. 30, 2009
Apr. 2020	Jul. 19, 2021	Apr. 2020	Sep. 30, 2020	Apr. 2020	Sep. 30, 2020
—	—	Jan. 1992	Jun. 30, 1992	Jan. 1992	Jul. 31, 1992
—	—	May. 2003	Nov. 30, 2003	May. 2003	Nov. 30, 2003

Notes: This table shows the business cycle troughs identified in real time by the MSQ(τ) models in Equation (3), as well as the vintage at which these trough dates could have been released. The table also contains the NBER trough dates and the dates these were released in real time. The real time identification strategy is discussed in Section 5.

Table 9
Real-Time Recession Identification: Ten-Year Simulations

Model	Number of Simulated True Peaks	Number of Correctly Identified Peaks	Number of Falsely Identified Peaks
	Average Across Simulations		
MSQ(25)	1.6	1.4	0.3
MSQ(50)	1.6	1.3	0.6

Notes: This table shows the average number of correct peaks and false peaks identified by the MSQ(τ) models in Equation (3) across 1000 simulations of ten-year sequences of monthly data vintages beyond July 2025. The real time identification strategy is discussed in Section 5.